Persuasion, Pandering, and Sequential Proposal*

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Abstract

I study a model in which an informed sender can propose a project to an uninformed receiver. The receiver can accept or reject the project's implementation. If the receiver rejects, the sender can propose a different project to the receiver, which, in turn, may be accepted or rejected. Overall, only one project can be implemented. Both players share preferences about the features of the project. Across projects, preferences are not aligned. There are two classes of equilibria. There always is a mixed-strategy equilibrium in which the sender panders towards the receiver-preferred project and the sender-preferred project is only implemented with a probability less than one. If the time horizon is sufficiently long a second type of equilibrium exists in which the sender persuades by waiting. In principle, many waiting equilibria exist. The shortest waiting equilibrium is ex-ante Pareto-dominated by the mixed strategy equilibrium, but a waiting equilibrium with intermediate waiting time is preferred by the receiver to all other equilibria. As an application, I consider a firm that needs clearance of a proposed merger by an anti-trust authority. Merger realizations are private to the firm. Both players prefer higher synergies. The authority prefers high competition, the firm prefers low.

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1 Introduction

An agent who implements a certain project often imposes an externality on the rest of the economy through implementation. Examples include mergers that change the market concentration, patent applications that limit competition, and research projects that require resources that otherwise are available for different purposes. To mitigate potential welfare losses caused by the project choices, agents are often required to apply to an authority who decides whether the proposed project should be implemented. In many cases, however, the authority is less informed than the agent about both, the project's realized quality itself, and the quality or existence of a potential alternative project. If private benefits of the project do not coincide with social benefits, the agent might have a strategic incentive to withhold a project for private benefits. Seeing an application, the authority must therefore answer the following questions: How large is the likelihood that an even better project exists, but is not proposed by the agent? To what extend is it worth denying the current proposal in hope for a better proposal tomorrow?

The aim of this paper is to study dynamic project choice in a sender-receiver framework. The sender has multiple rounds to propose projects to the receiver, but only one project can be implemented in total. The receiver, in turn, can only implement projects if they have been proposed. Thus, all the receiver can use as a signal for the quality of both the proposed and the not proposed projects is the choice of the sender's proposal. That means the receiver might reject a proposal in hope for a better proposal tomorrow, even if she expects the current proposal to be beneficial.

I show that in principle two types of equilibria arise: the first is a mixed-strategy equilibrium in which the authority accepts the receiver's less preferred project only with some probability. Similar to a related, static model by Che, Dessein, and Kartik (2013) the sender panders towards the receiver-preferred project as she expects this to be implemented with a larger probability. However, the dynamic setting allows for a second type of equilibria, if the time horizon is large enough. These are equilibria in which the sender persuades by waiting to propose the sender-preferred project. The set of waiting equilibria is potentially large. The waiting equilibrium with the shortest waiting time yields the same payoff for the sender, but a lower payoff for the receiver than the mixed strategy equilibrium. The receiver's profits are highest in a waiting equilibrium with intermediate waiting time. The reason is that in such a case good realizations of the sender-preferred project are implemented, but all others are deterred by the waiting period. As the waiting time increases also good realizations are deterred and as the waiting time shrinks worse realizations are implemented. Both is not beneficial to the receiver.

I contribute to the understanding of project choice problems in many environments in that I show that choosing an equilibrium with considerable delay is to the benefit of the receiver, while the sender never prefers any delay. This may be important in setting up institutions that might determine which type of equilibrium is played in reality. Authorities prefer to pick equilibria that involve considerable delay while private parties always prefer a quick solution.

As an application, I consider a firm (firm 0) within an oligopoly that proposes a merger with another firm to an antitrust authority. Upon seeing only the proposed merger, but not the realized synergies, the authority needs to decide whether to allow the merger or to block it. Even if the merger is preferred to the status quo, the authority faces the trade-off whether to allow the proposed merger or to block it in hope that firm 0 proposes an alternative merger with another firm in the next period. Different from Nocke and Whinston (2010), I consider non-disjoint mergers which is why a myopic policy is not optimal in this setting. I show that the implemented merger depends on the choice of equilibrium.

The analysis is motivated by two observations: first, a merger proposed by a given firm has typically two components: the merging partner and a realization of synergies. Second, if a merger is denied, a firm can offer an alternative merger in the subsequent period.

The two dimensions of a given merger proposal are particularly interesting as preferences are aligned in one dimension, but orthogonal in the other. Preferences about post-merger marginal cost are aligned: both the firm and the authority prefer less costly projects. Across projects, however, preferences differ: while the firm favors less competition post-merger, the authority wants to keep the level of competition as high as possible.

The dynamic component becomes particular interesting if synergies are non-verifiable. Then the merging firms can only commit on their identities, but not on the particular realization. The authority, however, observing the merging firms identities can only form beliefs about the merger's quality based on the merger choice by the proposing firm. Without commitment power this means that the authority updates, both her beliefs on the proposed merger as well as on the non-proposed mergers before making a decision whether to accept or reject the merger.

Although both, firm and authority, discount future payoffs in the same way, increasing the time needed to implement the firm-preferred project is to the advantage of the authority. If the equilibrium is chosen such that decisions are always made early in the process, the firm is willing to propose her preferred project whenever synergies are not far worse than in the authority-preferred project. Different to that, an equilibrium that involves a longer waiting time for the firm-preferred merger results in the firm proposing the authority-preferred merger more often. As the authority-preferred merger can be allowed right at the beginning, the time loss is mitigated for the authority. However, any equilibrium with a very long waiting time is neither preferred by the authority: on the one hand, the firm would, of course, always propose the authority-preferred merger if available. On the other hand, she would also do so in case the synergies in this merger are very low, but those in the firm-preferred merger are very high. As authority and firm would prefer the same implementation in such a scenario scenario, both would prefer an equilibrium with intermediate waiting time.

The remainder of this paper is structured as follows: in Section 2, I give an overview on the related literature. Section 3 introduces the general model and the solution concept. In Section 4, I derive equilibria for the case of two possible projects, and analyze the differences. Section 5 concludes.

2 Literature Review

The paper by Che, Dessein, and Kartik (2013) is probably the closest to mine. Similar to my model, they are interested in a game between an informed sender and an uniformed receiver. In their model, however, the receiver can in principle decide freely what to do, even without the proposal of the sender. While this assumption is reasonable in case a decision maker hires an expert to tell her what to do, my model is more concerned about an authority that regulates what firms are allowed to do. In many situations authorities have only the power to block certain actions, but cannot enforce a particular action of the firm. The pandering equilibrium in their model has similar characteristics to the mixed strategy equilibrium in my model. I show, however, that adding a dynamic component also allows for a second class of equilibria, which are worthwhile to study, as the authority is actually better off in some equilibria within this class.

As proposals are needed, my model is not a model of cheap talk per se. However, given the multidimensionality of the project realizations, I relate to a model by Chakraborty and Harbaugh (2010) who introduce multidimensionality into the cheap talk literature. They show, that multidimensional cheap talk often leads to full revelation. The major difference to this model is that senders in Chakraborty and Harbaugh (2010) can choose in which dimension to communicate to the receiver, while the dimension is fixed in my model. Moreover, they, too, rely on the fact that the receiver has all the decision power and can implement whatever product she wants.

Different to the literature on sequential delegation, e.g. Kovác and Krähmer (2013), who show that sequential delegation supersedes if the differences in preference is small, I do not have gradual information arrival in my model, but a fixed multidimensional type space.

This model also contributes to the old literature on multidimensional signaling of Quinzii and Rochet (1985), Wilson (1985), and Milgrom and Roberts (1986). While Wilson (1985) and Milgrom and Roberts (1986) consider one dimensional types with multidimensional signals, my model has multidimensional types, as e.g. Quinzii and Rochet (1985), but only a one-dimensional signal. Moreover, I consider a dynamic environment which is why, even without explicit cost of signaling, the firm is equipped with a signaling motive.

Since producing the signal itself is costless, this paper also relates to dynamic bargaining models with one-sided incomplete information going back to Sobel and Takahashi (1983). In line with that Fudenberg and Tirole (1983) and Admati and Perry (1987) formulated a theory which uses time as a strategic variable. While Sobel and Takahashi (1983) and Fudenberg and Tirole (1983) have a similar negotiation structure to mine, that is one party makes an offer, the other can only accept or reject and the game continues upon rejection, Admati and Perry (1987) show that strategic delay can be used to signal one's type in two sided-bargaining problems. The main difference to the bargaining literature is that the signal space in this paper is reduced to the identity of the project and payments are not allowed. Firms want to signal that whatever they propose is "without alternative" which, if credible, would lead to immediate implementation. Thus, the firm tries not so much to signal anything about the value of the current project, but tries to discourage the authority's hope for a better project in the future.

In the application part, the model closest to mine is that of Nocke and Whinston (2013). They, model a delegated choice problem to derive optimal merger control. The difference is that is, despite their model being static, that the firm can verify her synergies in the proposed merger. Thus, the only private information the firm has is about the characteristics of the *not proposed* mergers. Further, Nocke and Whinston (2013) assume full commitment of the authority, which is not assumed in my model.

Asymmetric information between firm and authority has been studied also by Besanko and Spulber (1993) who were probably the first to model merger control explicitly as a game between firms and authorities.[1] Ottaviani and Wickelgren (2011) introduce a choice of timing on the side of the authority and derive conditions when ex-post merger control is better than ex-ante merger control. Dynamic merger review is studied by Nocke and Whinston (2010) and Sørgard (2009). Both papers look at disjoint mergers and how the decision on proposing the merger depends on the approval rule of the authority. Sørgard (2009) derives a rule that determines an optimal investigation probability, while Nocke and Whinston (2010) consider sequential proposal. They show that a myopic policy is optimal if mergers are disjoint. The

[1]A small literature on merger remedies also connects to this paper, as "projects" can also be seen as different types of remedies. Papers that asses remedies before are, e.g. Lyons and Medvedev (2007), Cosnita and Tropeano (2009), Vasconcelos (2010), and Cosnita-Langlais and Tropeano (2012).

main difference to my model is, that mergers are not disjoint in my model. Much to the contrary, I look at the different merger options by on single firm and ask how this effects the authorities decision. I find, different to the setup of Nocke and Whinston (2010) that a myopic policy is not optimal in such a case. In fact I show, that the authority can actually use the time dimension to screen merger realization.

3 The General Model

This section gives a formal description of the game to be played.

3.1 Setup

Consider a game of two players, a sender (the firm) and a receiver (the AA). The firm can choose one among a set of projects to recommend to the AA by sending message m, and the AA can accept or reject the proposed project and a proposed project only. The number of projects is N. The realisation of each project can be one of the following

1. with probability $1 > \lambda_i > 0$ project i is not available,[2] or
2. with probability $1 - \lambda_i$ project i is available. If project i is available, its realisation c_i follows a random variable. I assume each c_i to be independently drawn from a distribution F_i with a density f_i that is continuous on its support $[\underline{c}, \overline{c}]$.[3]

The firm receives a payoff of $\delta^{t-1}\pi_i(c_i)$ if project i is implemented in period t, the AA receives a payoff of $\delta^{t-1}w_i(c_i)$.

I assume that the functions are ordered such that $\infty > \pi_1(c) > \pi_2(c) > ... > \pi_N(c) > 0$ and $w_1(c) < w_2(c) < .... < w_n(c) < \infty$ for any $c \in [\underline{c}, \overline{c}]$.[4] Each $\pi_i,\ w_i$ is twice continuously differentiable on $[\underline{c}, \overline{c}]$ and decreasing in its argument.

The game has a total number of T periods and both players discount with factor $\delta < 1$. The value of the status quo is normalised to 0.

[2]Note: The non-availability probability subsumes not only the cases in which a certain project is not available in the usual sense (in a merger context this could be due to personal problems between CEOs, ...). Moreover non-availability may also occur if the project is neither profitable for the firm nor the AA (for example if synergies are too low). In such a case the project never gets proposed and may therefore be treated as if it was not available at all.

[3]I assume common bounds for the ease of notation. None of the results depend on this if there is at least some overlap between the two.

[4]In the case of the merger framework, this reflects the firms preferences c.p. towards larger mergers (functions of lower cardinality) while the AA prefers smaller mergers (functions of higher cardinality). In addition, to reverse the preference order completely simplifies computations, but the general results do not depend too much on this particular preference order. All I need is that firm and AA do not share the exact same order of preferences.

For the sake of simplicity and to make the decisions non-trivial assume the following:

Assumption 1. Given it is the only project that is available, each project has, in expectation, a positive payoff to both players. In other words:

$$E[w_i] > 0 \quad \forall i.$$

Assumption 2. The probability distributions are such that

$$E[w_{n+1}] > E[w_n] \quad \forall n.$$

Assumption 3. Suppose the firm naively proposes the project that maximises its payoff under full acceptance. Then, the distributions are such that there exists at least one case in which the AAs best response is to wait for a better project in the next round. More formally, this means:

$$\exists j \neq i \in \{1, 2, ..., N\} : E[\delta w_j | \pi_i \in \max_{k \in N} \pi_k] > E[w_i | \pi_i \in \max_{k \in N} \pi_k].$$

Assumption 1 is only to facilitate computations. It serves to ensure that the threat of non-availability of other projects is present for any project i. Assumption 2, too, is for simplicity. It ensures that the AA not only pointwise prefers projects with higher cardinality, but prefers them in expectation, too. Both assumptions can be relaxed to milder versions, in which they only hold in conditional expectations. Finally, the last assumption is made to make the problem interesting. It is, thus, crucial to the problem. If assumption 3 was not fulfilled, preferences would be completely aligned, and the solution to the problem would be trivial.

All, but the realisations of each project is common knowledge.

The timing of the game is as follows:

1. Before the beginning of the first period, the firm privately observes the vector of realisations $\psi = (c_1, c_2, ..., c_N)$. I may refer to the vector also as the firm's type. The firm's type remains constant throughout the game.

   To deal with the non-availability of certain projects, assume that if a project was not available (which as noted above happens with probability λ_i), the entry in ψ is some number $\bar{\bar{c}} > \bar{c}$.

2. At the beginning of each period, the firm can propose

   (a) nothing (by posting identity "0")

(b) exactly one project identity that

i. is available, i.e. where $c_i \neq \overline{\overline{c}}$ and[5]

ii. has not been proposed in any previous period.[6]

3. At the end of each period, after observing a non-"0"-proposal, the AA can

(a) accept the proposal: in this case we arrive at a terminal node, the proposed project gets implemented and payoffs realize

(b) deny the proposal: in this case the game advances to the next period.

4. After the firm has proposed "0", the game continues independently of AA's action.[7]

I am looking for perfect Bayesian Nash equilibria in which no player plays a weakly dominated strategy.[8]

3.2 Action and Strategies

The firm's choice is a function that determines its proposal at each time t. This function is a mapping from the set of possible types ψ defined as Ψ to some message m_t. That is,

$$m_t : \Psi \mapsto M_t(\Psi, H_{t-1}).$$

By the rules of the game the set of available messages M_t depends both on the type of the firm (since non-available states are excluded) and the history (since re-proposal is excluded). Thus, at time t and for a given state ψ the set of available messages, $M_t(\psi)$, is defined as

$$M_1(\psi) = \{i : c_i \neq \overline{\overline{c}}\} \cup 0$$
$$M_{t+1}(\psi) = M_t(\psi) \setminus m_t \cup 0 \quad \forall t \geq 0.$$

[5]Again this assumption is made to simplify the argument. An alternative way to model reduced form unavailability would be to assume that an unavailable project has a negative payoff to firm and authority. This way, if the AA accepts the proposal with positive probability, the firm would never propose it or withdraw her offer after it has been made, which is essentially the same as proposing 0. To avoid unnecessary notation, I simply assume such a proposal is not possible.

[6]This assumption reduces the set of possible equilibria. Allowing re-proposal might cause equilibria in which the firm persistently proposes the same project. All equilibria I derive in this setting here, easily survive in a world were re-proposal is possible. Moreover, the set of equilibria I derive here without re-proposal survives certain refinements, which they survive in the re-proposal setup as well.

[7]A variant in which this is not the case is studied in section 4.1.

[8]Since this is a two player game, this restriction is equivalent to using an strategic form trembling-hand refinement.

Thus, the firm can only post an available project identity that has not been proposed before (i.e. a positive number) or "0" (the Null-Project) which means that it does not wish to implement anything.

The strategy of the AA is slightly more complex and involves a function that determines the acceptance probability and a function of beliefs over types.

Before taking an action, the AA forms a belief about the type of the firm. The belief function takes the following form

$$\beta_t : H_{t-1} \mapsto \Delta(\Psi),$$

where $H_t = \{m_1, ..., m_t\}$ is an ordered set of received messages[9] and $\Delta(\Psi)$ is the set of probability distributions over the possible states.

The AA chooses further a function ρ_t in each period to determine the acceptance probability after having received proposal m_t. That is

$$\rho_t : M_0^{max} \setminus H_{t-1} \mapsto [0, 1].$$

where M_0^{max}, the maximal message space at the beginning of the game, is simply the vector $\{1, ..., N\}$ since in principle each project has a positive probability of being available and thus in the message set. After each round, the AA can remove all elements out of the message set that already have been proposed since we excluded re-proposal.

3.3 Equilibrium Description

A configuration $\left\{\{m^*\}_{t=1}^T \ , \ \{\rho_t^*\}_{t=1}^T \ , \ \{\beta_t^*\}_{t=1}^T\right\}$ is a trembling-hand perfect Bayesian Nash equilibrium if the following conditions hold:

- m_t^* is chosen out of a set of best responses to the equilibrium probability function of the AA. This set of best responses, $M^*(\psi, \rho_t^*, H_{t-1})$ is described as

$$M^*(\psi, \rho_t^*, H_{t-1}) = \{i \in M_t : V\left(i, \psi, \rho_t^*, H_{t-1}\right) \geq V(j, \psi, \rho_t^*, H_{t-1}) \quad \forall j \in M_t\} \qquad (*)$$

  where $V(i, \psi, \rho_t^*, H_{t-1})$ is the value of proposing project i in period t after having

[9] Formally, to be precise we need to define the order $\succeq^H$ as $m_i \succeq^H m_j \Leftrightarrow i > j$ and $\tilde{H}_t$ as the set of all messages send up to point t. $H_t(\tilde{H}_t, \succeq^H)$ is then the function that describes this ordered set. For the ease of notation I am going to suppress the arguments $\tilde{H}_t$ and $\succeq^H$.

proposed history H_{t-1} in the past and acting optimally thereafter.[10]

To put some structure on $V(i, \psi, \rho^*, H_{t-1})$ it is useful to distinguish between two types of value functions:

i. $V(i, \cdot)$ denotes the *beginning of period value function*, that is the value of proposing project i *at the beginning of a period* and acting optimally thereafter.

ii. $\tilde{V}(i, \cdot)$ is the *end of period value function*, that is, it describes the value the choice of i has on all future periods given the firm acts optimally after that particular period.

Note that $\tilde{V}$ is only an auxiliary construction to properly describe the V, since for message m_t it holds that $\tilde{V}(m_t, \rho^*_{t+1}, \psi, H_{t-1}) = \delta V(m^*_{t+1}, \rho^*_{t+1}, \psi, H_{t-1} \cup m_t)$.

$\tilde{V}$ serves the simple purpose to spell out the *beginning of period value function*:[11]

$$V(i, \cdot) = \rho^*_t(i)\pi_i(c_i) + (1 - \rho^*_t(i))\, \tilde{V}(i, \cdot).$$

- The equilibrium probability function ρ^*_t is chosen such that, for all messages sent with a ex-ante positive probability, it holds that

$$\rho^*_t(m_t) \in \begin{cases} 0 & \text{if } E_t[w_{m_t}|H_t] < \delta\Upsilon\left(H_t\right) \\ 1 & \text{if } E_t[w_{m_t}|H_t] > \delta\Upsilon\left(H_t\right) \\ [0,1] & \text{else} \end{cases} . \qquad (**)$$

  Where $E_t[\cdot|H_t]$ denotes the (rational) expectations of the AA in period t conditional on a history of messages H_t, that is already including m_t.

  Further, $\Upsilon\left(H_t\right)$ is the expected value at the beginning of the next period conditional on rejecting this periods offer and acting optimally thereafter.

- Beliefs β^*_t are consistent with Bayes' rule whenever possible.

- No player plays a weakly dominated strategy.

4 Equilibrium for N=2

For the sake of tractability, I am considering the case of only two projects here.

[10] Note that the time dependency of the decision is already incorporated in the variables ρ_t and H_{t-1} such that the value function it self can be modelled as time independent.

[11] For the ease of notation it is, without loss of generality, assumed that $\rho^*_t(0) := 0$.

4.1 Equilibrium of the modified game

Before looking at the general game, I want to focus on a slightly modified version for the ease of exposition.

Definition 1. A game is called "modified" if it follows the setup as it was laid out in section 3 except for the additional rule that the game ends directly whenever the firm proposes 0, i.e. the "null-project".[12]

This modification is primarily of didactic use as it reduces the number of equilibria. Later, I am going to show that the result also survives in the general game even under quite demanding refinement criteria and discuss other equilibria (and there relation to this one) that exist only in the general game.

The modified game with only two possible projects has a simple type space, namely a vector $\psi = (c_1, c_2)$, and a message set in the first period that is at most $\{0, 1, 2\}$. Under assumption 1 and 3, proposing naively cannot result in an equilibrium. The AA would never accept project 1 and thus proposing 1 when 2 is also available is not optimal. If $\lambda_2 > 0$, rejecting 1 always and accepting 2 can also not be an equilibrium if we do not allow for weakly dominated strategies. Since we ruled out weakly dominated strategies, the firm would always propose project 1 if it is the only project available and under assumption 1 the AA should accept it. Moreover, it is even possible to state the following.

Lemma 1. *For the case of $N = 2$ and under assumption 1 and 3, no pure strategy trembling-hand perfect equilibrium exists in the modified game.*

The proof of this lemma can be found in appendix A, as can all others not in the text.

The intuition behind this lemma is straight forward. Proposing "0" is always weakly dominated if any other proposal is available since a proposal of "0" necessarily terminates the game and keeps the status quo, which is never preferred by the firm if any project is available. Thus the firm is always going to propose something. Assumption 3 rules out that the AA just waives whatever project is proposed. Since with some probability each project is not available, it cannot be optimal to unconditionally block a certain project either. The firm best responds to such a blocking strategy by proposing the project only if it is the only one available. With this strategy of the firm, however, unconditional blocking is not optimal since each project is profitable in isolation (assumption 1). Thus, pure strategies played by the AA can never be optimal.

[12] An equivalent formulation would be a setting in which the authority can shut down negotiations by formally "accepting" the null project and implementing it. Again, the equilibrium proposed in this section would survive such a game.

Since every finite game has at least one trembling-hand-perfect Nash equilibrium,[13] the trembling-hand-perfect equilibria must involve at least one mixing party. In fact, since project realizations are continuously distributed, the AA is the only player that is going to mix on more than a probability zero set. To construct the equilibrium, first recall the equilibrium conditions from equation $(*)$ and equation $(**)$. For the equilibrium construction assume that the second project is the conditionally better looking one, i.e.

Assumption 4. $E[w_1|\pi_1 > \pi_2] < \delta E[w_2|\pi_1 > \pi_2]$.

Although it puts some restriction on the distributions, this assumption still allows for a wide range of distributions. Whenever both state variables c_1, c_2 are identically distributed or if c_2 does not have too much weight on high cost draws we always can find a $\delta < 1$ such that the assumption is fulfilled. Further assume that project 2 is always accepted.[14]

Under these assumptions, the probability $q_\rho(\psi)$ of proposing project 1 for a given acceptance probability $\rho_1(1) = \rho$ is characterised by

$$q_\rho(\psi) = \begin{cases} 0 & \text{if} \quad \rho\pi_1(c_1) \leq \zeta\pi_2(c_2) \\ 1 & \text{if} \quad \rho\pi_1(c_1) > \zeta\pi_2(c_2) \end{cases} \tag{1}$$

with

$$\zeta = (1 - \delta + \rho\delta) \in [0, 1],$$

which accounts for both the alternative proposal and the second round. In equation (1) we may assume that the firm does not mix since indifference only happens on a probability zero set of ψ.

With the help of equation (1), it is possible to construct a cut-off function $\tilde{c}_\rho(c_2)$ that assigns, given ρ and c_2, a unique value $\tilde{c}$ such that for all $c_1 < \tilde{c}$ the firm prefers to propose project 1 while she is going to propose project 2 whenever $c_1 > \tilde{c}$. The function takes the following form:

$$\tilde{c}_\rho(c_2) := \begin{cases} \max\{c_1 : \rho\pi_1(c_1) \geq \zeta\pi_2(c_2)\} & \text{if it is not } \emptyset \\ \overline{c} & \text{if } c_2 = \overline{\overline{c}} \\ \underline{c} & \text{else.} \end{cases} \tag{$*'$}$$

Note that the function $(*')$ is defined for each $\rho > 0$ over $c_2 \in [\underline{c}, \overline{c}] \cup \overline{\overline{c}}$ and has the following properties:

i. it is continuous in ρ and c_2, since π_2 is continuous;

[13] The proof of this is due to Selten (1975) and can be found in various textbooks such as Fudenberg and Tirole (1991) and Mas-Colell, Whinston, and Green (1995).

[14] Later, I show that in equilibrium this is indeed the case.

ii. it is weakly increasing in c_2 since $\pi_1(c_1)$ and $\pi_2(c_2)$ are both decreasing in their arguments;

iii. it is weakly increasing in ρ since the left-hand side of the inequality increases in ρ and $\pi_1(c)$ decreases in c;

iv. for fixed ρ it is weakly increasing in δ since the right-hand side decreases in δ, giving the inequality (weakly) more slack at all points.

v. as $\rho \to 0$, $\tilde{c}_\rho \to \underline{c}$ for all c_2. Since $\pi_2(\overline{c}) > 0$ for all c_2 there exists an $\underline{\varepsilon}_{c_2} > 0$ such that $\forall\ \rho < \underline{\varepsilon}_{c_2} \Rightarrow \tilde{c}_\rho(c_2) = \underline{c}$. Monotonicity of $\pi_2(c_2)$ ensures that $\underline{\varepsilon}_{c_2}$ is decreasing in c_2;

vi. since $\pi_1 > \pi_2$ by definition, it holds that there also exists an $\overline{\rho} < 1$ such that for all $\rho > \overline{\rho}$ we have $\tilde{c}_\rho(c_2) \geq c_2$;

vii. as a consequence of the monotonicity in both arguments, it also holds that the smallest c_2 at which the highest $\tilde{c}_\rho(c_2)$ is reached[15] (weakly) decreases in ρ. $\tilde{c}(\underline{c})$, on the other hand, increases in ρ.

To define the equilibrium action of the AA, recall equation $(**)$. By Lemma 1 we have excluded pure strategy equilibria, thus, $1 > \rho > 0$.

To re-write condition $(**)$, define 1_2 to be the event in which project 1 is recommended although project 2 is available, too, and define 1_0 to be the event at which 1 is sent and it is the only message other than 0 that is feasible. Since condition $(*')$ defines the optimal behaviour of the firm for any ρ and therefore determines the probabilities and expectations of events 1_2 and 1_0, it is sufficient to rewrite condition $(**)$ as follows

$$\begin{aligned} E[w_1|m_1 = 1] &= E[\delta w_2|m_1 = 1] \\ \Leftrightarrow \quad (1-\tilde{\lambda}_2)E[w_1|1_2] + \tilde{\lambda}_2 E[w_1|1_0] &= (1-\tilde{\lambda}_2)E[\delta w_2|1_2] \\ \Leftrightarrow \quad (1-\tilde{\lambda}_2)E[w_1|1_2] + \tilde{\lambda}_2 E[w_1] &= (1-\tilde{\lambda}_2)E[\delta w_2|1_2] \\ \Leftrightarrow \quad (1-\tilde{\lambda}_2)\left(E[w_1 - \delta w_2|1_2]\right) + \tilde{\lambda}_2 E[w_1] &= 0, \end{aligned} \tag{$**'$}$$

where $\tilde{\lambda}_2 = \frac{\lambda_2}{\lambda_2 + (1-\lambda_2)P(m=1|1_2)}$.

The first step divides expectations into two subsets. The second then makes use of the fact that, given that project 1, but not project 2, is available, the firm would always propose

[15] Typically this highest $\tilde{c}_\rho(c_2) = \overline{c}$, but one may choose a ρ small enough that even $\tilde{c}_\rho(\overline{c}) < \overline{c}$.

1 in order to not play a weakly dominated strategy. Noticing that w_2 and w_1 in the first term are conditioning on the same event gives the third step of equation $(**')$.

Lemma 2. *If assumption 1 and 4 hold, there always exists a* ρ *such that* $1 > \rho > 0$ *and equation* $(**')$ *are fulfilled.*

Intuitively, this lemma says that it is always possible to choose a ρ so low that whenever both projects are available, the firm chooses the second project. In turn, this choice means that the firm only (weakly) prefers to offer project 1 if nothing else is available. If it did so whenever only 1 was available, the AA would want to accept that proposal since it is welfare-increasing in expectations. Since gradual changes in ρ lead at most to a gradual change in both expectations, the difference of the two is continuous. If the difference is continuous, the indifference condition must, in line with the intermediate value theorem, eventually be satisfied for some $1 > \rho > 0$.

In fact, it is possible to derive an even stronger statement

Lemma 3. *If the conditions for Lemma 2 hold, then whatever* ρ *solves equation* $(**')$ *is unique.*

The intuition to this lemma is, again, quite simple. As ρ increases, the firm increases the number of states in which it proposes project 1. For any given c_2, this increase leads to additional states that are all worse than the worst state under the original regime. Therefore, the expected payoff the AA earns from implementing project 1 can only fall in ρ for proposals of 1. The expectations of postponing implementation, on the other hand, increase for the same reasons, since there is a larger set of "desirable" c_2 that lead to a proposal of 1. Thus, increasing ρ decreases the expected payoff when implementing 1 and increases the expected payoff of waiting for any ρ. Consequently equation $(**')$ can at most hold for one $\rho \in (0, 1)$.

To describe the equilibrium, it remains to show that it is optimal to always accept project 2, i.e. $E[w_2|m = 2] \geq \delta E[w_1|m = 2]$ needs to hold under the equilibrium ρ. This is guaranteed by assumption 2, as the following lemma shows.

Lemma 4. *The AA prefers project 2 over project 1 whenever 2 is proposed.*

The intuition underlying this lemma is that, in expectation, whenever the firm proposes a certain project, this project needs to be "better" than a certain threshold. In turn, whenever the firm does not propose a project it must have a worse realization than this threshold. Since in equilibrium the AA is indifferent under the proposal of 1, the expected payoff from project 2's realization must be higher for the authority if the firm proposes 2. Expectations of waiting after the proposal of project 2 must be smaller by the same argument. Thus always accepting project 2 is optimal in equilibrium.

Combining Lemma 2 and 4 provides existence of a mixed strategy equilibrium as the following lemma shows

Lemma 5. *Suppose assumption 1, 2 and 4 hold. Then, there exists a unique mixed strategy trembling hand perfect Baysian Nash equilibrium in the modified game with the following properties:*

- *The second project is accepted whenever it is proposed.*
- *The first project is accepted with probability $\rho < 1$.*
- *If the second stage is reached, proposal project 2 in the second stage gets accepted.*
- *Firms propose project one whenever ψ is such that $c_1 < \tilde{c}_\rho(c_2)$.*

4.2 Comparison to the original game

All the results of the previous section have been derived under the assumption that players play the modified game. In this section, I am going to discuss how these results carry over to the original game.

Theorem 1. *There exists an equilibrium of the original game that has the following properties:*

- *It is outcome equivalent to the unique equilibrium of the modified game*
- *On the equilibrium path actions are identical to the ones of the modified game*
- *The equilibrium does not fail the universal divinity, if any of the two conditions holds*
  - *i. the acceptance probability ρ^* is larger than the discount factor δ, or*
  - *ii. the worst project is smaller than the outside option $w_1(\overline{c}) < 0$.*

The crucial aspect for survival of the refinement is to find beliefs such that the AA wishes to deny off equilibrium proposals. For the second proposal this is not that hard since the firm never wishes to deviate since that only would incur time costs. Thus, any type is equally likely to deviate. The story is quite different for project 1. There might be a reason to wait for the firm if it beliefs that in later periods the acceptance probability was large enough. For sure, any off-path acceptance probability that attracts at least some type $\hat{\psi}$ must also attract all types that cannot propose project 2 because it is unavailable. However, discriminating between those is not possible, since they are all only interested in implementing project

1. This way, if any type has, for some off-path acceptance probability any incentive to go off-path, also the type $\underline{\psi} = (\bar{c}, \bar{\bar{c}})$ has an incentive to do so. This is, by definition the worst type in terms of payoff for the AA. Thus, if there is at least one state such that the AA prefers the outside option, she prefers the outside option to implementing project 1 under $\underline{\psi}$. Since this type engages in all off-path activities, it cannot be excluded under universal divinity. Thus, if the AA beliefs any deviator is of type $\underline{\psi}$ she has reason to deny it and the equilibrium sustains.

The crucial aspect for the sustainability of the equilibrium is the fact that there exists a positive probability that the better looking project does not exist.

Together with the second restriction, namely that $w_1(\bar{c}) < 0$, the non-availability guarantees universal divinity for any length of the game. Thus, the existence of the equilibrium of the modified game carries over to the general game even if we were to restrict our self to a rather narrow set of equilibria.

Uniqueness, however, does not necessarily survive. In particular, the two dimensional type space of the firm allows for a wide range of equilibria if we consider the generalized game. Together with the possibility to sent the “0” message as often as possible a wide range of justifiable off-equilibrium beliefs can be consistent even under strict refinements such as universal divinity. To understand this recall that the equilibrium of the modified game does not fail universal divinity in the general game simply because “all” types for which project 2 was not available survive the iterated D2 criterion. That is, if the AA sees a proposal of project 1 anywhere off the equilibrium path it can, even with strong restrictions be of any type (regarding project 1’s cost function).

If negotiations go on for sufficiently many rounds, that is if T is large enough, a second set of equilibria which I am going to call “waiting equilibria” would arises. In such a “waiting equilibrium” the firm proposes the null-project for sufficiently many rounds in order to signal that the ex-ante AA-preferred project is of poor quality or not available. In the traditional bargaining literature these are sometimes the only equilibria that survive even mild refinements and they are typically called “signalling equilibria”.[16] In the present context signalling also is an issue in the “ρ^*-equilibrium” described above, which is why I differ in terminology.

The decision whether to accept or reject an offer made by the firm in those type of equilibria is in fact very similar too the decision rule in the ρ^*-equilibrium. Whenever the AA believes that the project offered in the current round is at least as good as the discounted value of what she can expect in the future, she would accept the proposed project, whenever this is not the case she denies approval. Given this, the firm might delay its proposal

[16] Examples would be Admati and Perry (1987) or Cramton (1992)

strategically under some circumstance while it may prefer to propose something that gets accepted right away instead of suffering the cost of waiting. More precisely, suppose the AA always accepts project 2 and accepts project 1 only at time $t \geq \tau + 1$.[17] Then for each c_2 it is possible to construct a function $\hat{c}_\tau(c_2)$ such that the firm chooses to propose project 1 in period τ whenever $c_1 < \hat{c}_\tau(c_2)$ and project 2 (if possible) in all other cases. More formally this is

$$\hat{c}_\rho(c_2) := \begin{cases} \max\{c_1 : \delta^\tau \pi_1(c_1) \geq \pi_2(c_2)\} & \text{if it is not } \emptyset \\ \overline{c} & \text{if } c_2 = \overline{\overline{c}} \\ \underline{c} & \text{else.} \end{cases} \tag{$*'$}$$

This function almost perfectly corresponds to $\tilde{c}^*_\rho(c_2)$ defined in the beginning of this section. Consequently, this, too, provides a cutoff value for the firm given any τ. As in the other equilibrium the AA optimally chooses τ such that whenever the firm proposes project 1 in period $\tau + 1$ it holds that the AA only accepts the proposal if $E_{\tau+1}[w_1|H_{\tau+1}] \geq \delta E_{\tau+1}[w_2|H_{\tau+1}]$. Before describing the possible "waiting equilibria" let us first properly define the term.

Definition 2. An equilibrium is called a "waiting equilibrium at τ" if it is an equilibrium of the game and the authority uses the following strategy:

- Project 2 is accepted in any period
- Project 1 is rejected in any period $t \leq \tau$ and accepted in all other periods.

Next, it is straightforward to use the cutoff function $\hat{c}_\tau$ derived above to describe the equilibrium that is closest related to the ρ^*-equilibrium from above.

Lemma 6. *Consider a certain specification of the game and fix the equilibrium at ρ^*. Ignoring the integer constraint in t there exists a "waiting equilibrium at $\underline{\tau}$" where*

$$\underline{\tau} = \frac{\ln(\rho^*) - \ln(1 - \delta(1 - \rho^*))}{\ln(\delta)}$$

that corresponds in the firms decision rule to the ρ^*-equilibrium, that is $\hat{c}_{\underline{\tau}}(c_2) = \tilde{c}_{\rho^*}(c_2)$ for all c_2. This equilibrium may be considered as the "shortest possible" waiting equilibrium in a sense that all other waiting equilibria require a waiting time $\tau \geq \underline{\tau}$. Moreover, if and only if $\rho^* > \frac{\delta}{\delta+1}$, then $\underline{\tau} < 1$.

[17] To understand why this is $\tau + 1$,recall that a proposal in t=2 is discounted with δ^1 since participants ex-ante payoffs are given in period one values.

The intuition behind this becomes quite clear if you think about what happens if the AA accepts proposals of project 1 "too early". Then the firm would propose them in a way, such that the AA better denies it in hope of a better outcome on the other project. From the construction of the ρ^*-equilibrium it is already clear that whenever the firm decides via cutoff rules, then there exists a unique decision function such that the AA is indifferent in expectations between the two projects. While this function was enforced in the ρ^*-equilibrium with the acceptance probability, it is now done by appropriately choosing τ. Since the two correspond we just need to look for the parameter in which the functions are identical. This is what describes the smallest τ.

In the model, of course, time is discrete and thus the above described point is generically never an integer. However, as the following proposition shows, for all $\tau > \underline{\tau}$ a waiting equilibrium at τ exists.

Theorem 2. *If $T-1 > \underline{\tau}$, there exists a set of waiting equilibria at τ for all τ such that $\underline{\tau} \leq \tau \leq T-1$. All waiting equilibria survive under the refinement of universal divinity if $w_1(\bar{c}) < 0$*

While each equilibrium on its own is in fact independent of the maximum duration of the game, there is a one to one mapping between numbers of proposal rounds played after $\underline{\tau}$ and the number of waiting equilibria that exist.

4.3 Welfare effects

To compare the different equilibria it is useful to think about how the equilibrium payoffs in the different equilibria behave and to exercise some comparative statics. Following the order in which the equilibria were characterized, I start by examining the ρ^*-equilibrium and thereafter looking at the waiting-equilibria.

Besides comparing the equilibria with each other, a natural benchmark would also be the (ex ante) equilibrium payoffs if the AA had the ability to commit ex-ante to accepting only project 2. This benchmark case is going to be called a "conservative AA" throughout the rest of the paper.

To simplify notation denote $E_a[w|\rho^*]$ to be the ex-ante payoff expectation of the AA under equilibrium acceptance probability ρ^* as in the equilibrium of section 4.1 and let $E_a[w|0]$ be the ex-ante expectations if the AA was to deny project 1 always, i.e. behaving conservatively.

Proposition 1. *The ρ^*-equilibrium always assigns a lower payoff to the AA than she gets when committing to only accepting the second project.*

Proof. The mixed strategy equilibrium increases the AA's payoff only if $E_a[w|\rho^*] > E_a[w|0]$. The following argues that this is not true.

Spelling out the first yields

$$E_a[w|\rho^*] = (1-\lambda_2)(1-\lambda_1)\int_{\underline{c}}^{\bar{c}} w_2(c_2)\left(1 - F_1(\tilde{c}_{\rho^*}(c_2))\right) \mathrm{d}F_2(c_2) \tag{2}$$

$$+ (1-\lambda_2)\lambda_1 E[w_2] \tag{3}$$

$$+ (1-\lambda_2)(1-\lambda_1)\int_{\underline{c}}^{\bar{c}} \delta w_2(c_2) F_1(\tilde{c}_{\rho^*}(c_2)) \mathrm{d}F_2(c_2), \tag{4}$$

where the first is the expected value in case both projects are available and project 2 is proposed given ρ^*, the second is the part in which project 2 is proposed since it is the only one available, and the last part covers the expectations whenever 1 is proposed, using the indifference condition ($**'$).

Further we may decompose

$$\begin{aligned}
E_a[w|0] =& (1-\lambda_2)E[w_2] \\
=& (1-\lambda_2)\left(\lambda_1 E[w_2] + (1-\lambda_1)E_[w_2]\right) \\
=& (1-\lambda_2)\lambda_1 E[w_2] + (1-\lambda_2)(1-\lambda_1)\int_{\underline{c}}^{\bar{c}} w_2(c_2) dF_2(c_2) \\
=& (1-\lambda_2)\lambda_1 E[w_2] \\
& + (1-\lambda_2)(1-\lambda_1)\int_{\underline{c}}^{\bar{c}} w_2(c_2)\left(F_1(\tilde{c}_{\rho^*}(c_2) + (1 - F_1(\tilde{c}_{\rho^*}(c_2)))\right) dF_2(c_2) \\
=& E_a[w|\rho^*] + \underbrace{(1-\delta)(1-\lambda_2)(1-\lambda_1)\int_{\underline{c}}^{\bar{c}} w_2(c_2) F_1(\tilde{c}_{\rho^*}(c_2)) \mathrm{d}F_2(c_2)}_{\geq 0}.
\end{aligned} \tag{5}$$

The first step just spells out the conditional expectations, the second divides it into the events in which project one is or is not available. The third transforms them into integral form, the fourth multiplies with 1 to back out the ex-ante expectations of the equilibrium in the last step. Since expectations in equilibrium are positive when 1 is proposed, the leftover term is positive. Thus, from an ex-ante point of view the AA prefers accepting only project two if both were available. □

Intuitively, this is rather obvious if one thinks about the different channels at work in the model.

If the AA was able to allow only project 2 the firm can never exploit the game in her preferred way if opinions about optimal actions differ. Thus, the persuasion channel is completely shut down. Pandering, to the contrary, is driven to its maximum.[18] Finally, the sequentiality channel is shut down as well, since in equilibrium there is no second period. This sequentiality works, as we saw in the discussion of the equilibrium, as an insurance for the firm to "try out" her preferred project. That way, commitment to pure strategies in fact must work in favour of the AA since it only shuts down the channels that benefit the firm. The AA therefore earns something like a "conservative commitment markup". To facilitate later comparison it is useful to think of this markup as relative to the expected payoff if only project 2 was was proposed. To do so, define

$$\phi(\rho^*) := \frac{E[w|0] - E[w|\rho^*]}{E[w|0]}$$

and rewrite equation (5) as

$$E_a[w|0] = (1 - \phi(\rho^*))E_a[w|0] + \phi(\rho^*)E_a[w|0]. \tag{6}$$

In this context ϕ can be interpreted as the "relative conservative commitment markup" (RCCM).

With this reformulations it is easier to tell how much (in relative terms) the AA looses by being unable to ignore any proposals of 1 in equilibrium.

Proposition 2. *Consider the ρ^*-equilibrium. An increase in the time preference parameter δ leads to*

- *a decrease in the acceptance probability ρ^*,*
- *a decrease in the ex-ante likelihood that project 1 is being proposed and*
- *a decrease of the RCCM (and a higher absolute payoff for the AA).*

While an increase in δ decreases the cost of waiting one more round for both the firm and the AA, the increased expected welfare in the second period leads the AA to reduce its probability of accepting the first project. That means that the firm reduces the states at which it proposes the first project and panders more towards the second. This way it needs

[18] In fact ex-ante pandering is optimal from the point of view of the AA. However, on an interim stage there might actually be over-pandering, that is the firm proposes a project that both firm and AA prefer less than the other one (interim), but that was not preferred ex-ante by the AA.

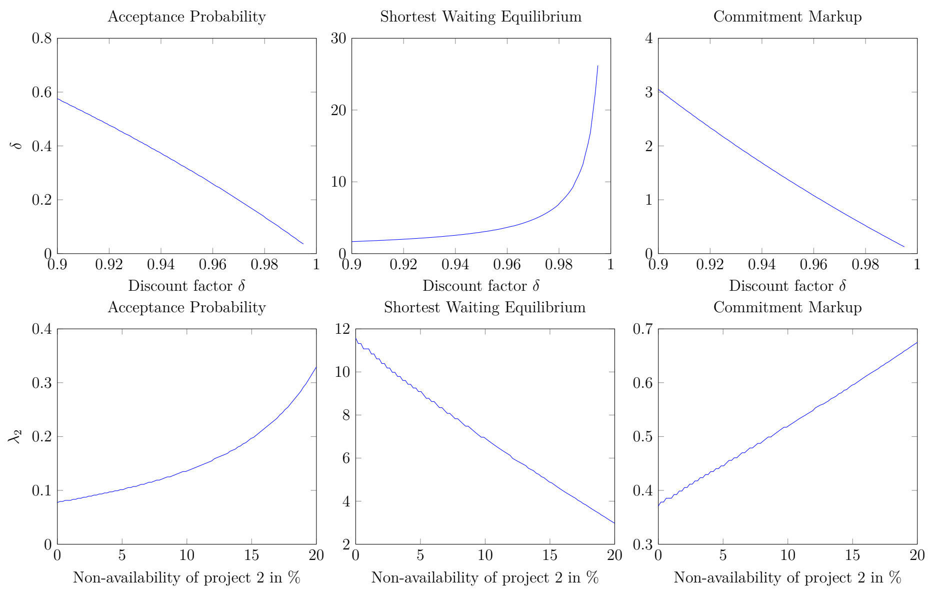

Figure 1: Comparative statics with respect to availability of the second project λ_2 and discount factor δ. The upper panels deal depict effects on the ρ^*-equilibrium and the lower panels that on the shortest waiting equilibrium.

to give up some of the gains from the increased δ. Further, the "commitment markup" goes down and thus, the AA loses less from not being able to commit to a pure strategy.[19]

Another interesting comparative statics in this model is the effect of non-availability of the second project. This sheds light on to the point of how much the threat of not having an alternative projects works in favour of the firm.

Lemma 7. *Consider the mixed strategy equilibrium ρ^*. An increase in the non availability probability of project 2, λ_2, leads to*

- *an increase in the acceptance probability ρ^* in equilibrium*
- *an increase in the ex-ante likelihood that project 1 is being proposed and*
- *an increase of the RCCM.*

This result is, of course, not very surprising. As it becomes more likely that the second project is not available at all, the firm creates, by proposing project 1, a larger threat to the AA that this is the only project it has in fact. Thus, given project 1 has been proposed, there is a higher chance that it is the only one around (in which case the AA would want to accept) and the interim expectations of the AA are driven down. She reacts by increasing the acceptance probability.

[19]In section 4.4 I discuss why commitment to a pure strategy is not optimal either and why this leads to lower credibility of a commitment assumption on the AA's side.

In terms of the AAs payoffs under no commitment, to the contrary, the story is not so clear since λ_2 effects both the ex-ante expectations of a conservative strategy as well as those under the equilibrium strategy.

The relative mark-up increases nevertheless, as the interim pressure the firm can put on the AA increases with a higher probability of project one being the only one available.

Turning to the waiting equilibria, it might at first be interesting to see how the set of possible waiting equilibria changes with the parameters. While the largest waiting equilibrium (if any such equilibrium exists) is always defined by T, as Theorem 2 shows, the shortest waiting equilibrium might change as we change the parameters of the model. With help of Lemma 6 it is obvious that all parameters except the discount factor δ effect $\underline{\tau}$ only through ρ^*. Observe, that

$$\frac{\partial \underline{\tau}}{\partial \rho^*} = \frac{1}{ln(\delta)\rho^*} \, \frac{1-\delta}{1-\delta(1-\rho^*)} < 0.$$

Thus, as ρ^* increases $\underline{\tau}$ decreases. The reason can easily be found in the equivalence of the decisions rules of the firm in both equilibria. Therefore, a higher effective acceptance probability needs to result in a smaller minimum waiting period.[20]

The only parameter, that effects $\underline{\tau}$ also in another way than through ρ^*, is the discount factor δ. Its effect on $\underline{\tau}$ is not all that obvious since the direct effect could go either way. Nonetheless, as the following lemma shows, the overall effect of a change in δ on $\underline{\tau}$ has the opposite sign than a change of δ has on ρ^*.

Lemma 8. *Consider a waiting equilbrium at $\underline{\tau}$. An increase in the discount factor δ leads to an increase in $\underline{\tau}$.*

Even if the effect though ρ^* always supersedes the direct effect of δ, the additional effect that a change in the discount factor has already points towards the presence of some difference in the analysis of the waiting equilibria compared to that of the ρ^*-equilibrium. These differences are what is going to be considered next. To do so, I especially focus on two effects not present in the ρ^*-equilibrium, that may, in addition, help to understand the change in payoff between the different waiting equilibria.

First, each round of waiting decreases the AAs payoff since the agreement is reached later in time. The AA is impatient and therefore suffers “cost of delay”. Second, and quite

[20] In fact, in terms of elasticity, one can show that the movements do not correspond one-to-one but that $\underline{\tau}$ shrinks less than proportional compared to the increase in ρ^* if $\delta > \rho^*$ since that implies

$$0 > \frac{\partial \underline{\tau}}{\partial \rho^*} \frac{\rho^*}{\underline{\tau}} > -1.$$

different is what I call the “benefit of the doubt” effect which works in the other direction. If in equilibrium there is a longer waiting period, this does not only lead to pandering, but also to a smaller set of states for which the firm decides to wait. That means the more costly it is to wait for the firm, the more often she proposes the AA-preferred project already in the first round and thus the (potential) delay is to the benefit of the AA.

Which effect dominates, if we were to move from the waiting equilibrium at τ to the one at $\tau + 1$ is hard to say without further parameter restrictions. Ignoring again, the integer constraint in t and assuming that T is large enough, the “AA-most-preferred” equilibrium (in an ex-ante sense) lies in the interior of the interval $(\underline{\tau}, T-1)$. In fact the shortest waiting equilibria, i.e. that at $\underline{\tau}$ is never the most preferred one as it is always dominated by the ρ^*-equilibrium. For T sufficiently large, the most preferred equilibrium is in addition also neither the ρ^*-equilibrium nor an equilibrium in which for all possible states at most project 2 gets implemented as the following proposition shows:

Proposition 3. *Consider a game in which*

$$T - 1 > \overline{\tau} = \frac{\ln \pi_2(\overline{c}) - \ln \pi_1(\underline{c})}{\ln \delta}.$$

Then, ignoring the integer constraint in t, the “AA-most-preferred equilibrium” is a waiting equilibrium at τ^* with $\tau^* \in (\underline{\tau}, T-1)$. Moreover, independent of the length of the game, the “AA-most-preferred equilibrium” is never at $\underline{\tau}$.

Further description of the AA-most-preferred waiting equilibrium is, however, hard to accomplish since there are counteracting effects in the derivative of the expected payoff of the AA. To see this, it may be helpful to disentangle these expectations:

$$\begin{aligned} E[w|\tau] = & \underbrace{(1-\lambda_1)(1-\lambda_2) \int_{\underline{c}}^{\overline{c}} w_2(c_2) \left(1 - F_1\left(\hat{c}_\tau(c_2)\right)\right) \mathrm{d}F_2(c_2)}_{A} \\ & + \underbrace{\lambda_1(1-\lambda_2)E[w_2]}_{B} \\ & + \underbrace{(1-\lambda_1)(1-\lambda_2)\delta^\tau \int_{\underline{c}}^{\overline{c}} \int_{\underline{c}}^{\hat{c}_\tau(c_2)} w_1(c_1)\mathrm{d}F_1(c_1)\mathrm{d}F_2(c_2)}_{C} \\ & + \underbrace{\lambda_1(1-\lambda_2)\delta^\tau E[w_1]}_{D}. \end{aligned}$$

Terms A and B describe the expectations in cases in which the firm decides to propose project 2 right away. C and D describe them in cases in which project 1 is proposed after the

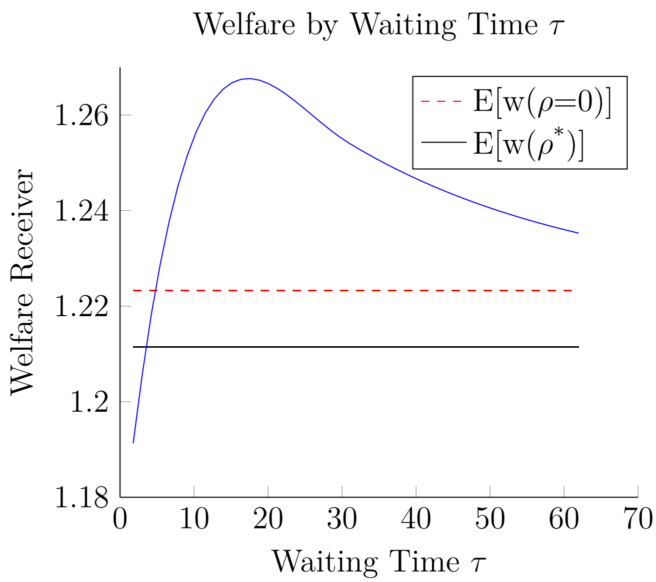

Figure 2: Welfare effects in a linear demand Cournot economy for mergers from three firms to two. The blue line is the (ex-ante) payoff of the AA in a waiting equilibrium at τ. It always starts below the black solid line (by assumption 4) and is above the dashed line and decreasing to the right of $\overline{\tau}$. For comparison the black solid line is the (ex-ante)payoff of the ρ^* equilibrium and the red dashed line denotes the (ex-ante) payoff to the AA if she could (again ex-ante) commit to reject any proposal of the larger merger. Although single-peakness of the blue line is assured in simple linear-demand Cournot models, it cannot be generalized to more complex settings.

waiting time. Within the two blocks, the first term (i.e. A and C) describes the situation in which both projects are available, the second (B and D) the case in which the proposed project is the only one available.

Observe that term B is in fact completely independent of τ and D decreases in τ. The other two terms are not that easy to determine, so let us look at the derivatives a bit closer

$$\frac{\partial A}{\partial \tau} = (1-\lambda_1)(1-\lambda_2)\int_{\underline{c}}^{\overline{c}} w_2(c_2)\left(1-f_1\left(\hat{c}_\tau(c_2)\right)\right)\frac{\partial \hat{c}_\tau(c_2)}{\partial \tau}\mathrm{d}F_2(c_2) \tag{7}$$

$$\begin{aligned}\frac{\partial C}{\partial \tau} &= (1-\lambda_1)(1-\lambda_2)\ln(\delta)\delta^\tau \int_{\underline{c}}^{\overline{c}}\int_{\underline{c}}^{\hat{c}_\tau(c_2)} w_1(c_1)\mathrm{d}F_1(c_1)\mathrm{d}F_2(c_2) \\ &\quad + (1-\lambda_1)(1-\lambda_2)\delta^\tau \int_{\underline{c}}^{\overline{c}} w_1(\hat{c}_\tau(c_2))f_1(\hat{c}_\tau(c_2))\frac{\partial \hat{c}_\tau(c_2)}{\partial \tau}\mathrm{d}F_2(c_2)\end{aligned} \tag{8}$$

While the sign of the first derivative and the second part of the second derivative remain unidentified due to the fact that w_i can take on different values, the first part of the second derivative is clearly negative. However, the results of Proposition 3 indicate that at some point $\frac{\partial A}{\partial \tau}$ in combination with the second part of $\frac{\partial C}{\partial \tau}$ must overturn the negative effects in the rest of the terms. Without further parameter restrictions, it is not possible to pin down when this overturning is going to take place or even to decide whether the $E[w|\tau]$ function is single peaked in τ between $\underline{\tau}$ and $\overline{\tau}$.

4.4 The case of commitment

In many application it is assumed that authorities such as an AA have the possibility to ex-ante commit themselves to a certain probability rule. This is often justified by the idea that an authority is a long lasting institution that cares for reputation and therefore has the opportunity to credibly commit itself to a certain action. As shown above, a commitment to a pure strategy is ex-ante preferred by the AA if she finds herself in the ρ^*-equilibrium. In this part I am going to argue that such a commitment cannot be optimal. Thus, the AA would need to commit to a mixed strategy. This, in contrary is in practice again hard to implement.[21]

In terms of the structure of the game, notice that allowing for commitment changes the following to aspects:

(i) With commitment the game changes in a way such that the AA is now a first mover.

[21] One reason might be, that (real) randomization across proposals might violate legal statues. However, if the AA needs to justify denials, she might have an incentive not to stick to the commited randomization rule and thus acts in an interim optimal way as in my model.

(ii) With commitment the AA can only gain compared to the case without commitment.

The second statement is not only a corollary to Proposition 1, but in such settings an even more general point. If commitment is an option the AA could always replicate the outcome of the game as if it was played without commitment by simply committing to some ρ or τ as needed in any equilibrium. Since it has no private information at all, the firm would, given the probabilities, propose exactly as in the case without commitment. This way any commitment device can only be to the benefit of the AA.

Another important fact, that is useful for constructing a solution to the commitment game is the following

Remark. A commitment game can be modelled w.l.o.g. ignoring any beliefs either of the players have.

To see this, observe that the only private information present here is the state vector ψ. If we let nature move after the AA has chosen their probability rule (which by definition only depends on common knowledge parameters, i.e. the prior), then there is never any uncertainty about the state in this game and we can restrict ourselves to (trembling-hand) subgame perfect equilibria.

Suppose now, the AA would commit to a rule where she accepts project 1 with probability 0 and project 2 with probability 1. In such a case the expected welfare was

$$(1 - \lambda_2)E[w_2], \tag{9}$$

since the firm always proposes the second project whenever it is available.

However, a pure strategy as described above is not optimal in the commitment case as the next proposition shows.

Proposition 4. *It cannot be profitable for the AA to commit to a rule of pure time-invariant strategies in which she always accepts one of the projects and always rejects the other (in each period). it is neither optimal to accept both projects in the first period nor to accept none.*

This proposition is strongly connected to Proposition 3. There it is already stated that the best waiting equilibrium yields a higher payoff than commitment to a pure time-invariant decision rule would.

One might now wonder whether commitment to a pure strategy is possible in which the firm accepts a certain project only after some time period much like the waiting equilibria. In fact this can also never be optimal as the following proposition shows:

Proposition 5. *In a game with commitment it is never optimal for the AA to commit to a strategy where she accepts one project right away and another after a certain time t has past.*

Proof. Recall the decision of the firm in any waiting equilibrium. For any τ it is possible to find a $\rho < 1$ such that

$$\delta^{\tau} = \frac{\rho}{1 - \delta(1 - \rho)}.$$

Under such ρ it holds that $\hat{c}_{\tau} = \tilde{c}_{\rho}$ by definition. But whenever the firm proposes 1 under τ she does so under ρ as well, but earlier than under ρ. Thus, the cost of delay shrinks, yielding a higher payoff to the AA. $\square$

In other words, pure strategies are never optimal with commitment.[22] But that might lead to some "strategic trembling" as for example to accept project 1 once it has been proposed, since it might be the only one available. Afterwards, to maintain credibility, an institution would of course claim this only happened by "mistake" or has an even easier job by blaming the randomization device if we allow for commitment to mixed strategies. Thus, avoiding interim information updates of the AA might be a hard job even for an authority and not too easy to assume in this setting.

To sum up, the commitment option might serve as a benchmark, but should not be taken too seriously when it comes to implementation. If at all, the AA could impose a fixed waiting time to achieve at least second best and to use this as a coordination device to pick her most preferred equilibrium.

5 Concluding Remarks

I consider a dynamic sender-receiver game in which an informed firm can propose a project to an uniformed authority who decides whether to implement the project or to block it. A proposal is required to implement a project and the firm may propose an alternative project in case her proposal is rejected. Overall, only one project can be implemented.

In principle there a two types of equilibria: There is a mixed-strategy equilibrium in which the authority implements her less-preferred project with a certain probability already in the first period, and a set of waiting equilibria in which the authority implements it only after a certain period of waiting. A firm that postpones her proposal signals a high quality of the authority's less-preferred project. The authority's most-preferred project, on the other hand, is always implemented right away in both types of equilibria.

[22] Note that we ruled out trivial examples with assumption 4.

I find that both firm and authority prefer the mixed-strategy equilibrium over a short waiting equilibrium to save on the cost of delay. However, as the time of waiting increases, intermediate realizations of the authority's less-preferred projects are deterred from being proposed. This has a positive effect on the authorities ex-ante expected payoff. If the waiting time becomes too long, however, all realization of the authority's less-preferred project are deterred which decreases the authorities payoff. The sender never prefers a waiting equilibrium over the mixed-strategy equilibrium.

My findings show that multiple periods allow authorities to learn from the proposal of the firm and yields better solution than the static model from the authority's point of view. The reason is that the firm is in competition with her future self. The improvement materializes either through *pandering* by the firm such that she sometimes proposes a less-preferred project to avoid it being blocked. Alternatively, the firm may *persuade* the authority by delaying her proposal to signal a good realization of the firm-preferred project.

I show that firms always prefer a quick implementation of a project, while authorities prefer to delay the implementation of an ex-ante less-preferred project for an intermediate length. Delaying the implementation for too long is not beneficial for the authority, but still preferred to mixing at the initial period. A delay time that is too short on the other hand is not preferred by either of the two players.

The findings contribute to the discussion on how approval processes should be designed in a dynamic setting. I show that delaying certain approval decisions can be more effective in screening projects than probabilistic acceptance rules. Such waiting games are, however, not to the benefit of the firm who has an incentive to lobby for an equilibrium that leads to instantaneous proposals at any point. Thus, I provide a theory that suggests that delays in the merger review process might not entirely be determined by technical constraints. Instead, there may be a strategic component for delay depending on the choice of equilibrium. Moreover, although possible, an authority would not want to switch to an equilibrium with shorter waiting time as this harms her ex-ante expected payoff.

The insights gained in this analysis provide several interesting directions for future research. While I focus on a game theoretic approach in this paper, a natural follow-up is to consider a mechanism design approach instead. Close to the analysis here would be a mechanism without commitment power. That is a third-party that collects the multi-dimensional private value of the firm, i.e. the realizations of all projects available, and offers a recommendation to both firm and authority on how to proceed. This may overcome parts of the coordination failure in the mixed-strategy equilibrium and makes this equilibrium more attractive for the authority. A second possible direction is to allow the authority to costly investigate the proposal of the firm. If she had some technology to test any efficiency claims, e.g. via a mean-preserving spread as in Kamenica and Gentzkow (2011), at an interim stage, she may have an incentive to do so if the alternative project is likely to be similar in terms of quality. On the other hand, if she expects the alternative to be considerably different, she might not want to bear these cost. Finally, results may be affected if there was competition on the firms side, that is if two firms compete about whose project is going to be implemented. In such a case firms have a stronger incentive to pander towards the authority preferred project, as they do not compete only against their future selves, but also against a competitor within each period.

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A Appendix: Proofs

A.1 Proof of Lemma 1

Proof. The proof proceeds by contradiction.

In general, in any pure strategy equilibrium, it needs to hold for the acceptance vector $\boldsymbol{\rho}_1 = (\rho_t^1, \rho_t^2)$ that $\rho_t^i \in \{0, 1\}$ for all i and t. In this case, both i and t need only to be considered within $\{1, 2\}$.[23]

In a first step, I show that if least one project is profitable in the second stage, there is no pure strategy equilibrium. The second step considers then the remaining case.

First Step

Define $E_t[w_i|j]$ the expectations of the authority at time t about project i if project j has been proposed in t.[24]

For an equilibrium in pure strategies in the subgame of the second period, it must be the case that $\rho_2(i) = 1$ if $E_1[w_i|j] > 0$ for $j \neq i$. Suppose this was the case.

Then, for a pure strategy equilibrium of the whole game, $\rho_1(j) = 1$ if $E_1[w_j|j] > \delta E_1[w_i|j] > 0$. Again suppose this was the case.

If all this was true, the Firm for sure proposes j if $\pi_j > \pi_i$ and whenever only project j is available. Assumption 3 implies that $\rho_1(i) = 0$, since otherwise the firm would propose i whenever it maximises its profits.

This implies that for all cases in which both $c_i \neq \bar{\bar{c}}$ and $c_j \neq \bar{\bar{c}}$, the firm proposes j.

If $c_j = \bar{\bar{c}}$ but $c_i \neq \bar{\bar{c}}$, the firm must propose i because proposing 0 is a weakly dominated strategy. In fact this is the only state at which the firm proposes i, but then, by assumption 1, $E_1[w_i|i] > 0 = E_1[w_2|i] \Rightarrow \rho_1^i = 1$, a contradiction.

Thus, whenever assumption 1 and 3 hold and $E_1[w_i|j] > 0$, it cannot be that $\rho_1(k) = 1$ for $k \in \{i, j\}$.

This means the only possibility left in this case is to deny all proposals in the first period.

With this, it is not possible that $\rho_2(k) = 1$ for both k since that would lead the firm to propose j whenever i is more profitable and the other way around. But by assumption 3 the AA would then accept at least one proposal. This violates the assumption that $\rho_1(k) = 0$ for both k and is a contradiction.

In turn, this means at least one merger needs to be denied even in the second round. Let this be merger j. As a result, whenever j is available, j is proposed and gets denied in the first stage for trembling-hand-perfection. But then again, if only i was available, this would be proposed for sure in period 1 due to the refinement and the assumption of the modified game. As discussed above the AA would then need to accept such a proposal since she knows that j does not exist. This violates $\rho_1(i) = 0$ and is, again, a contradiction.

Second Step

To complete the proof we need to consider the case in which $E[w_i|j] < 0$ for both $i = 1, 2$. Suppose this was the case, then we know that $\rho_2(k) = 0$ for any k. As we know from the

[23]Note that if the game comes to a third period, then the firm must propose 0 along the way which ends the modified game immediately.

[24]Note again by the rules of the modified game and $N = 2$ the tuple (j, t) suffices for a full history of proposals.

previous discussion $\rho_1 = 1$ cannot be true, thus at least one merger needs to be declined in the first stage.

If only one gets denied in the first period, the same arguments as in step 1 hold. If the firm knows merger j gets rejected in both stages, while i gets accepted only in the first stage, it proposes i whenever possible in the first period. However, if only j is around it is going to be proposed. But then j must be accepted by the AA.

The remaining pure strategy equilibrium candidate is now that no project gets accepted at any stage. Thus, $E_t[w_i|i] < 0$ for all i and t. This again cannot be achieved if assumption 1 holds and the firm never chooses a weakly dominated strategy.

□

A.2 Proof of Lemma 2

Proof. Notice first that equation ($**'$) is continuous in ρ since $\tilde{c}_\rho$ is continuous. Notice further that by assumption 4 for $\rho = 1$ it holds that $E[w_1|m_1 = 1] < \delta E[w_2|m = 1]$ conditional on the firm best responding. Now pick some $\rho' > 0$ such that

$$q_{\rho'}(\psi) = 0 \quad \forall \psi : c_1, c_2 \neq \bar{\bar{c}},$$

which exists since $\pi_1(\bar{c}),\ \pi_2(\underline{c}) > 0$. Choosing such a ρ' is equivalent to setting $\tilde{c}_{\rho'} = \underline{c}$. The first part of the last line in equation ($**'$) reduces to 0 with $\tilde{c}_{\rho'}(c_2)$. The latter part, however, does not depend on it and is, by assumption 1, positive. Thus, the left hand side of the last line of equation ($**'$) can be both positive and negative within the relevant range and due to continuity and the intermediate value theorem, this means that equation ($**'$) also holds for at least one $1 > \rho > 0$. □

A.3 Proof of Lemma 3

Proof. To prove uniqueness first observer from the second line of equation ($**'$) that $\lambda_2 E[w_1]$ is constant over all $0 < \rho < 1$. To show that ρ is unique it suffices thus to show that $h(\rho) := E[w_1|1_2] - \delta E[w_2|1_2]$ is decreasing in ρ. I do so in two steps. The first proofs that $E[w_1|1_2]$ is in fact decreasing in ρ and the second is to show that $E[w_2|1_2]$ is increasing in ρ. This suffices to conclude that $h(\rho)$ is decreasing in ρ and thus, ρ is unique.

First Step To see that $E[w_1|1_2]$ is decreasing in ρ fix some $1 > \rho > 0$ and c_2.

Given this observe that the firm proposes project 1 if $c_1 \leq \tilde{c}_\rho(c_2)$. Suppose now that ρ was increased to $1 \geq \rho' > \rho > 0$. By the properties of $\tilde{c}_\rho$ we know that $\tilde{c}_{\rho'} \geq \tilde{c}_\rho$. This in turn means that there exists a (possibly empty) interval $[\tilde{c}_\rho, \tilde{c}_{\rho'}]$ for which the firm proposes project 1 under ρ' but not under ρ. Whenever project 1 was proposed under ρ, it is also proposed under ρ'. Since w_1 is a decreasing function in c this means that $E[w_1|c_1 \leq \tilde{c}_{\rho'}, c_2] \leq E[w_1|c_1 \leq \tilde{c}_\rho, c_2]$ for all c_2. Integrating over all c_2 this leads to the following statement:

$$E[w_1|c_1 \leq \tilde{c}_{\rho'}] \leq E[w_1|c_1 \leq \tilde{c}_\rho],$$

and thus $E[w_1|1_2]$ is decreasing in ρ.

Second Step To see now that $E[w_2|1_2]$ is increasing in ρ fix again some $1 > \rho > 0$ and in addition c_1.

Define now

$$\hat{c}_2 := \begin{cases} \min[\tilde{c}^{-1}(c_1)], & \text{if it exists} \\ \underline{c}, & \text{otherwise.} \end{cases} \tag{10}$$

Then the firm proposes 1 for every draw $c_2 \geq \hat{c}_2$. If we now pick $1 \geq \rho' > \rho > 0$, the firm would propose 1 under the new regime whenever it has been proposed under the old regime. In addition due to the $\tilde{c}_\rho$ being increasing in ρ there exists an additional (possibly empty) interval $[\check{c}_2, \hat{c}_2]$ for which the firm proposes project 1 as well. Since $\check{c}_2 \leq \hat{c}_2$ this leads to (weakly) increasing expectations in the decreasing function w_2. Thus,

$$E[w_2|c_2 \geq \hat{c}_2] \geq E[w_2|c_2 \geq \check{c}_2].$$

Since this holds for a generic c_1 we can similar to step one draw the conclusion that $E[w_2|1_2]$ increases in ρ, thus $h(\rho)$ is decreasing in ρ which completes the proof.

□

A.4 Proof of Lemma 4

Proof. The AA strictly prefers project 2 to project 1, whenever project 2 is proposed if and only if the following condition holds

$$E[w_2|m = 2] > \delta E[w_1|m = 2].$$

Claim. Suppose the firm plays its equilibrium strategy given any $0 < \rho < 1$. Then $E[w_1|m^* = 1] \geq E[w_1|m^* = 2]$.

To see this, fix some c_2 and ρ.

Recall that w_1 is a monotone decreasing function and observe that thus

$$E[w_1(c_1)|c_2, c_1 < \tilde{c}_\rho(c_2)] \geq E[w_1(c_1)|c_2, c_1 > \tilde{c}_\rho(c_2)] \tag{11}$$

since

$$\inf\{w_1(c_1) : c_1 < \tilde{c}_\rho(c_2)\} = w_1(\tilde{c}_\rho(c_2)) = \sup\{w_1(c_1) : c_1 > \tilde{c}_\rho(c_2)\}.$$

Since the relation in equation (11) holds for any c_2 it also holds that integrating out c_2 we have the following relation

$$\begin{aligned} & E[w_1(c_1)|c_1 < \tilde{c}_\rho(c_2)] \geq E[w_1(c_1)|c_1 > \tilde{c}_\rho(c_2)] \\ \Leftrightarrow \quad & E[w_1(c_1)|m^* = 1] \qquad \geq E[w_1(c_1)|m^* = 2] \end{aligned}$$

Note that for this we only used the decision rule of the firm and thus can by the same argument (just redefining the cut-off as a function of c_1) state that

$$E[w_2|m^* = 2] \geq E[w_2|m^* = 1].$$

Together with the equilibrium condition from equation ($**'$) this implies the following

$$E[w_2|m^* = 2] \geq E[w_2|m^* = 1] > \delta E[w_2|m^* = 1] = E[w_1|m^* = 1] \geq E[w_1|m^* = 2]$$

and thus,

$$E[w_2|m^* = 2] > \delta E[w_1|m^* = 2]. \tag{12}$$

□

A.5 Proof of Lemma 5

Proof. The first two properties directly come from Lemma 2 and 4 except for the part that $E[w_2|m = 2] > 0$. Uniqueness follows from Lemma 3.

For the remaining property, namely that project 2 gets accepted in the second period, observe that this only happens if project 1 is proposed in the first period. A proposal of 1 in $t = 1$ occurs if either 1 is the only available project or 1 and 2 are available, but 1 is more profitable to the firm given the AAs strategy. In the first event, the expected payoff is simply $E[w_1]$. For cases where both 1 and 2 are available, notice that this event implies that for every c_2 there exists a $\tilde{c}_\rho(c_2) \leq \bar{c}$. The firm only proposes 1 if $c_1 < \tilde{c}_\rho(c_2)$. Given any c_2 this implies $E[w_1|m = 1, c_2] \geq E[w_1] > 0$. Integrating over all c_2 can therefore not change the sign. Thus, $E[w_1|m = 1] = \delta E[w_2|m = 1] > E[w_1] > 0$ and the proposal in the second stage, project 2 is always accepted.

What is missing is now to show that $E[w_2|m = 2] \geq 0$. This holds since we already know from the above that $E[w_2|m = 1] > 0$. If this is the case, we also know that (similar to the discussion about $\tilde{c}_\rho(c_2)$) there exists a cut-off $\hat{c}(c_1)$ for each c_1 such that message project 2 is sent whenever $c_2 < \hat{c}(c_1)$. Since w_2 is decreasing in c it must be the case that $E[w_2|m = 2] \geq E[w_2|m = 1] \geq 0$. Thus, project 2 is not only preferred to project 1 if project 2 is proposed but it is also preferred to not accepting.

Trembling hand perfection of the overall game is easily checked, recognizing that players play no weakly dominated strategies if indifferent. Neither accepting nor rejecting is weakly dominated and so is no mixture of the two. The firm is only indifferent on a probability zero event, and even then there is no strategy that dominates the other. For all other cases, trembling of the AA would not change the result. For the outcome irrelevant subgame after 2 has been proposed and denied (which never occurs in equilibrium), we can take any of the trembling hand perfect equilibria in the subgame, which exist for sure.

The last part that is missing is to show that expectations are positive along the path. This is ensured by assumption 1 and the observation that project 1 needs to be sufficiently good to be proposed, thus its expectations cannot be negative. Since they equal the expected value of waiting this cannot be negative either. Lemma 4 then provides that the expectations of project 2 cannot be negative either. □

A.6 Proof of Theorem 1

Proof.

On the equilibrium path: First, observer that on the equilibrium path of the modified game, the AA accepts any proposal in the second round. Suppose now, that players play the same strategies in the original game, then there is no reason why in the second period either off the players should have an incentive to deviate from the strategies they played in the modified game. Taken this as given, the reduced form game of the first period leaves the same options for both players as the modified game. Except for the case in which the firm proposes 0, the game is of no difference from the perspective of the players as the reduced form game of the first period of the modified game. Thus, again there is no incentive to deviate from the equilibrium strategies of the modified game. Since 0 is only proposed if the firm has no project available in the modified game, the same is going to happen in the original game. This way, on the equilibrium path it is trivial that the firm would (for any history of 0) have only 0 in its message set and would thus propose 0 in every period until T. Thus, the actions of both players on the equilibrium path are identical to those of the modified game and the outcome is the same as that of the modified game. The same holds if the firm only has project 1 available and gets rejected. Then the firm proposes 0 in the second period and forever after until the termination period T.

Off the equilibrium path:

1. First consider the off equilibrium action in which the AA observes the firm proposing project 1 in the second period after a proposal of 0 in the first period. These are in terms of off equilibrium beliefs the most problematic ones.

   The proof of this part is of constructive nature and defines beliefs that are consistent with the notion of universal divinity[25] In each step I am going to describe which beliefs actually are in line with the criterion itself to thereafter argue why iterative application would not fail the criterion either.

   i. Assume $\rho^* > \delta$. Suppose further project 1 is available. This means that whenever the firm was to propose 1 on the equilibrium path, a necessary condition for a deviation with positive probability under the intuitive criterion (and therefore also necessary under D2) is

   $$\rho^*\pi_1 + (1-\rho^*)\delta\max\{\pi_2, 0\} \leq \delta\pi_1.$$

   This can be rearranged to

   $$(\rho^* - \delta)\pi_1 + (1-\rho^*)\delta\max\{\pi_2, 0\} \leq 0.$$

   The above cannot hold if $\rho^* > \delta$ and therefore whenever project 1 was proposed in the first round on the equilibrium path, it has a 0 probability to be proposed after a 0 is observed off the equilibrium path.

[25] This has been introduced by Banks and Sobel (1987).

If 2 was proposed in the first round it gets accepted for sure, thus a deviation into the second round can only be profitable for types where $\rho^* < \delta\rho_2'$, with $\rho_2' < 1$ being the second round acceptance probability for project 1. This necessity contradicts the assumption that $\rho^* > \delta$. Thus, also those types are eliminated under the intuitive criterion. Since no type survives the intuitive criterion, none thus survive D2. Thus, the equilibrium is robust to those refinements.

Iteration in this case is not in question, since all types are eliminated already after the first round.

ii. Assume that $w(\bar{c}) < 0$.

For what follows, it is necessary to define the set of best responses the AA has to such an off equilibrium deviation. In principle the AA can respond either with 0 or 1 and both responses can be supported by at least one belief system. Due to the continuous density, there also exists at least one convex combination of beliefs that justify a response of 0 and 1 respectively which makes the AA indifferent. thus, all (mixed) actions of the AA are part of the best response.

Next, for a given state $\psi = (c_1, c_2)$ with $c_i \neq \bar{\bar{c}}$ and the equilibrium ρ^* a deviation for the firm as proposed can only be profitable if the following condition holds:[26]

$$\max\{\pi_2(c_2), \rho^*\pi_1(c_1) + (1-\rho^*)\delta\pi_2(c_2)\} \leq \rho'\delta\pi_1(c_1) + (1-\rho')\delta^2\pi_2(c_2) \tag{13}$$

This can be rearranged to

$$\max\{(1-\delta^2+\rho'\delta^2)\pi_2(c_2), \rho^*\pi_1(c_1)+(1-\rho^*-\delta(1-\rho'))\delta\pi_2(c_2)\} \leq \rho'\delta\pi_1(c_1) \tag{14}$$

Fixing some arbitrary $c_1 = c$ and ρ' for which the firm wants to deviate at least in some states and propose 0 in the first and 1 in the second period. With this, the left hand side of condition 14 is increasing in π_2. For the first element this is trivial, since $(1-\delta+\rho'\delta) > 0$ by definition. For the second part, observe that the second element is increasing in π_2 if and only if

$$1-\rho^*-\delta(1-\rho') > 0.$$

Suppose now, this was not the case, i.e.

$$1-\rho^* \leq \delta(1-\rho').$$

Then, since $\delta < 1$, it must hold that $\rho' < \rho^*$. If that was the case, then a deviation is never profitable for any state since waiting for the second period actually would

[26]Note that I am assuming here, that project 2 gets accepted in the subsequent period. Assuming any kind of different acceptance probability for project 2 other than 0 would not change the result. Since this effect is independent of the parameter c_1 those types may not be excluded under certain belief systems. However, results (and passing the divinity criterion) does not depend on this at all.

yield a worse outcome even ignoring any time cost δ. Thus, the ρ' that has been chosen does not fulfil the requirement that at least someone wants to deviate.

However, for $\rho' > \rho^*$ the left hand side of condition (14), is increasing in π_2.

That, in turn implies, that if condition (14) would not hold for any $\psi = \{c, c^x\}$ it does not hold for any $\psi\{c, c_2 > c^x\}$ either.

Finally, consider the case where $\rho' = \frac{\rho^*}{\delta}$. Under this regime it holds that

$$\rho'\delta\pi_1(c_1) < \rho^*\pi_1(c_1) + (1 - \rho^* - \delta(1-\rho')\delta\pi_2(c_2),$$

but at the same time if project 2 was not available, it is true that

$$\rho'\delta\pi_1(c_1) = \rho^*\pi_1(c_1) \tag{15}$$

and thus, a deviation is (weakly) profitable.

Last, observe that in equation (15) the value of π_1 does not matter at all, i.e. whenever it is profitable to deviate for some state $\psi(\hat{c}, \overline{\overline{c}})$ deviations for all other states $\psi(c_1, \overline{\overline{c}})$, given $c_1 \neq \overline{\overline{c}}$ are also profitable for the agent.

With this, D2 eliminates all types accept for those where $c_2 = \overline{\overline{c}}$, and since this contemplates to the complete range of $c_1 \in [\underline{c}, \overline{c}]$ it is in fact possible for the firm to have any sort of believe that is consistent with D2, that is any believe that excludes project 2 from being existent. A belief of $\beta(m_1 = 1) = B(\psi)$, where $B(\psi)$ is a probability function with

$$B(\psi) = \begin{cases} 1 & \text{if } \psi \geq (\overline{c}, \overline{\overline{c}}) \\ 0 & \text{else,} \end{cases}$$

would be of such kind and justifies a rejection in the second period, whenever 0 was observed in the first period as long as $w_1(\overline{c}) < 0$.

To complete universal divinity for this type of deviation, notice, that the set of best responses does not change, as long as $w_1(\overline{c}) < 0$. Thus, no further elimination takes place and universal divinity is fulfilled for this type of deviation.

2. Second, consider deviations in which the firm after $l < T$ periods of proposing 0 suddenly proposes project 1.

   Since a deviation in case 1. fulfils the criteria of universal divinity, it mus also hold that any later period fulfils the criteria since the only thing that changes is the discount factor which is now δ^l. Since $0 < \delta^l < 1$ the same as in case 1. applies.[27]

3. Third, consider deviation in which the firm proposes 2 off the equilibrium path.

   This deviation is (due to the discount factor) never profitable for any state (given that the AA rejects 1 with probability 1 at all times except $t = 1$). Thus, the AA cannot infer any beliefs from the observed deviation and the equilibrium action is arbitrary

[27] An exception might be the terminal period. If $w_1(c_1) < 0$, there exists a reasonable belief that justifies $\rho_T = 0$ even in the terminal period and the argument goes through.

and we can (for the sake of simplicity) assume that 2 is always accepted after l rounds of 0s.

4. Finally, consider deviations in which the firm proposes 0 in period 2 after having proposed 1 in period 1.

   Then again, since on the equilibrium path the firm accepts 2 always, there is no possible gain from deviation and the AA cannot infer anything from it. Thus, we might as well assume whatever we want, e.g. that 2 is always accepted.

□

A.7 Proof of Lemma 6

Proof. The proof of existence is a straightforward adaptation of the results of the ρ^*-equilibrium. Observe that due to the two decision rules whenever

$$\delta^{\tau} = \frac{\rho}{1 - \delta(1 - \rho)}, \tag{16}$$

the function $\hat{c}_{\tau}$ is in fact identical to $\tilde{c}_{\rho^*}$. Thus, the decision rule of the firm is the same. Applying the ln on both sides of equation (16) yields the expression for $\underline{\tau}$. This constitutes an equilibrium only if the AA acts optimally, too. On path, this holds, since we know already from the ρ-equilibrium that $E[w_2|2] > \delta E[w_1|2]$ in the first period. Since the firm is going to propose 2 in the first period whenever $c_1 > \hat{c}_{\underline{\tau}}(c_2)$, all subsequent periods must be due to some trembling error the firm has made. It is always possible to find a totally mixed strategy sequence such that for $\epsilon \to 0$ the AA prefers to accept 2 whenever proposed.

If project 1 is proposed in any period prior to (and including) $\underline{\tau}$ this is an off path action and can be rejected by the AA if we assign beliefs that such early proposal of project 1 are often enough[28] made by types for which the second project is more beneficial to the firm. After period $\underline{\tau}$ the firm nonetheless proposes project 1 whenever $c_1 < \hat{c}_{\underline{\tau}}(c_2)$. As in the description of the ρ^*-equilibrium, this decision rule makes the AA indifferent at time $\underline{\tau}$ between accepting or rejecting the proposal. Thus, there is no incentive to deviate from the proposed strategy. Finally, for all subsequent periods anything gets accepted for example, if we assign beliefs to the AA that only "good enough" types make the mistake of proposing very late in the game.

To see that no equilibrium with less waiting time exists, fix the set $\tilde{\boldsymbol{\psi}}$ of possible states ψ in which the firm proposes project 1 in the first period of the ρ^*-equilibrium. This set is identical to the set $\hat{\boldsymbol{\psi}}_{\underline{\tau}}$ of states in which the firm chooses to wait. If the waiting need was reduced to $\tau < \underline{\tau}$, the set $\hat{\boldsymbol{\psi}}_{\tau} \supset \hat{\boldsymbol{\psi}}_{\underline{\tau}}$. Since $\delta < 0$ there exist some $0 < \rho' < 1$ such that

$$\delta^{\tau} = \frac{\rho'}{1 - \delta(1 - \rho')}$$

Since the left hand side decreases in τ and the right hand side increases in ρ, the corresponding $\rho' > \rho^*$. By the monotonicity arguments used in the proof of the ρ^*-equilibrium,

[28] In a sense that it is very likely that such off path behaviour comes from this type of firms.

this means that the AA is not indifferent any more between accepting the offer and delaying it another period. In fact, she prefers to wait. Thus, there exists an incentive to deviate on path and thus, there does not exist any shorter waiting equilibrium than $\underline{\tau}$.

The last part follows by simple algebra. $\underline{\tau} \leq 1$ is equivalent to

$$\delta \leq \frac{\rho^*}{1 - \delta(1 - \rho^*)}.$$

This statement is in turn equivalent to

$$\rho^* \geq \frac{\delta}{\delta + 1}$$

□

A.8 Proof of Theorem 2

Proof. Recall from Lemma 6 that under the decision rule that is described there, the AA is indifferent between accepting and rejecting the offer of the firm in $\underline{\tau}$. If the AA increases τ, monotonicity in $\hat{c}_\tau$ shrinks the set of states in which the firm chooses to wait. As in the proof of Lemma 3, "worse" states of project 1 are dropped earlier than "better" ones. Thus, the AA is not indifferent but in fact strictly prefers acceptance in period $\tau + 1$ under the firms decision rule $\hat{c}_\tau$ whenever $\tau > \underline{\tau}$. By the same reasoning, the firm accepts project 2 whenever it is proposed. By asuumption $E[w_2] > 0$. However, only if $\pi_2(c)$ is "bad enough" the firm chooses to wait. Thus, expectations are higher than $E[w_2]$ and positive whenever project 2 is proposed. Finally, for the same reason it never pays off for the AA to wait for a proposal of project 1 whenever the firm offers project 2. Thus, the AA has no incentive to deviate. The firm has neither, since she follows her optimal decision rule $\hat{c}_\tau$.

For universal divinity consider first the following equilibrium situation. The firm has some type $\psi = \{c_1, c_2\}$ for which it is optimal to wait for τ periods to propose project 1 in the $(\tau + 1)^{th}$ period. Consider now a deviation in which the firm proposes project 1 at an earlier stage, that is e.g. τ. The firm would benefit from this type of deviation whenever the acceptance probability of project 1 in period τ is such that $\delta^{\tau-1}\rho_\tau(1)\pi_1(c_1) > \delta^\tau \pi_1(c_1)$. Since this condition is in fact independent of c_1, all such types are equally likely to deviate and non can be excluded by universal divinity. The same reasoning obviously holds for any $\tau' < \tau$

Second, consider a type ψ' in which the firm chooses to propose project 2 in period 1 in equilibrium. When, would such a type choose to deviate and to propose project 1 in τ? The condition for this is a bit more involved and requires that $\delta^{\tau-1}\rho_\tau(1)\pi_1(c_1) > \pi_2(c_2)$. This is now of course not independent of the type any more, but due to the fact that the firm chose to propose project 2 in equilibrium, it must hold that $\delta^\tau \pi_1(c_1) < \pi_2(c_2)$. Combining the two, requires thus that $\delta^{\tau-1}\rho_\tau(1)\pi_1(c_1) > \delta^\tau \pi_1(c_1)$, but then no one has a greater desire to deviate to τ than all types that propose 1 at $t = \tau + 1$ in equilibrium and we can exclude all types that propose 2 in equilibrium. Since there is no further distinction possible, all surviving types survive any further iteration of this reasoning and cannot be separated. Thus, any type that chooses to wait in equilibrium can be part of the belief of the AA. If we, as in Theorem 1, assign a degenerate belief that the firms type is $\underline{\psi} = \{\bar{c}, \bar{\bar{c}}\}$ whenever a deviation is observed, then the firm has even under universal divinity no incentive to react any different than staying on the equilibrium path. With this belief, the AA denies all earlier off-equilibrium proposals and the equilibrium survives universal divinity. $\square$

A.9 Proof of Proposition 2

Proof. First, recall again the equilibrium condition as used in the proof of Lemma 3

$$\begin{aligned} \lambda_2 E[w_1] + (1-\lambda_2)h(\rho^*) &= 0 \\ \Leftrightarrow \lambda_2 \left(E[w_1] - h(\rho^*)\right) + h(\rho^*) &= 0 \end{aligned} \tag{17}$$

where

$$h(\rho^*) = E[w_1|1_2] - \delta E[w_2|1_2] = \frac{\int_{\underline{c}}^{\bar{c}} \int_{\underline{c}}^{\tilde{c}_{\rho^*}(c_2)} [w_1(c_1) - \delta w_2(c_2)] dF_1(c_1) dF_2(c_2)}{\int_{\underline{c}}^{\bar{c}} F_1(\tilde{c}_{\rho^*}(c_2)) \mathrm{d}F_2(c_2)}$$

where the first term is the expected value whenever only project one is available which is independent of δ and ρ. The $h(\rho)$ function, however, depends on δ both directly and indirectly via ρ^*.

Since equation (17) exists and holds for all $0 < \delta < 1$ and $0 < \lambda_2 < 1$ and by assumption 1 (unconditional expectations are positive), it holds that $h(\rho^*) < 0$.

Using that equation (17) must hold as δ changes, we can derive the following

$$\frac{\partial \lambda_2 E[w_1] + (1-\lambda_2)h(\rho^*)}{\partial \delta} = \tag{18}$$
$$(1-\lambda_2)\left[\underbrace{\frac{\partial h(\rho^*)}{\partial \delta}}_{<0} + \underbrace{\frac{\partial h(\rho^*)}{\partial \tilde{c}_{\rho^*}}}_{<0}\left(\underbrace{\frac{\partial \tilde{c}_{\rho^*}}{\partial \delta}}_{\geq 0} + \underbrace{\frac{\partial \tilde{c}_{\rho^*}}{\partial \rho^*}}_{\geq 0}\frac{\partial \rho^*}{\partial \delta}\right)\right] = 0.$$

The sign of the direct derivative can be seen by inspection, that of the derivative with respect to ρ has been shown in the proof of Lemma 3 and the discussion of $\tilde{c}_\rho(c_2)$. Recall further that for all c_2 for which the equality that describes $\tilde{c}_{\rho^*}$ is not binding a marginal change in ρ^* has no effect, while for all c_2 such that

$$\tilde{c}_{\rho^*}(c_2) = \max\left\{c_1 : \frac{\rho^*}{\zeta}\pi_1(c_1) = \pi_2(c_2)\right\}$$

$\tilde{c}^*_\rho$ is increasing in ρ^*. Since the acceptance probability is independent of c_2, we may conclude that

$$\frac{\mathrm{d}\tilde{c}_{\rho^*}(c_2)}{d\delta} = \frac{\partial\tilde{c}_{\rho^*}}{\partial\delta} + \frac{\partial\tilde{c}_{\rho^*}}{\partial\rho^*}\frac{\partial\rho^*}{\partial\delta} \leq 0$$

is necessary for equation (18) to hold and requires

$$\frac{\partial\rho^*}{\partial\delta} < 0.$$

Second, observe that the ex-ante probability that project 1 gets proposed is

$$(1-\lambda_1)\left(\lambda_2 + (1-\lambda_2)\int_{\underline{c}}^{\bar{c}} F_1(\tilde{c}_2)\,\mathrm{d}F_2(c_2)\right).$$

Taking the derivative with respect to δ yields

$$(1-\lambda_1)(1-\lambda_2)\int_{\underline{c}}^{\bar{c}} f_1(\tilde{c}_{\rho^*})\frac{\mathrm{d}\tilde{c}_\rho(c_2)}{d\delta}\,\mathrm{d}F_2(c_2)\ \leq 0,$$

since $f_1 \geq 0$ by definition and $\frac{\mathrm{d}\tilde{c}_\rho(c_2)}{d\delta}$ is negative for reasons given above.
Finally, recall from equation (5) that

$$\phi E[w|0] = (1-\delta)(1-\lambda_2)(1-\lambda_1)\int_{\underline{c}}^{\bar{c}} w_2(c_2)F_1(\tilde{c}_{\rho^*}(c_2))\mathrm{d}F_2(c_2).$$

The RHS decreases in δ both directly and indirectly via $\tilde{c}_{\rho^*}$.

Thus the RCCM goes down. Since $E[w|0]$ is independent of δ, $E[w|\rho^*]$ increases in absolute terms, too. □

A.10 Proof of Lemma 7

Proof. Similar to the proof of Proposition 2, we take the derivative of equation (17) and take the derivative with respect to λ_2, that is

$$\begin{aligned}\frac{\partial\lambda_2 E[w_1] + (1-\lambda_2)h(\rho^*)}{\partial\lambda_2} &= \\ E[w_1] - h(\rho^*) + (1-\lambda_2)\left[\frac{\partial h(\rho^*)}{\partial\tilde{c}_{\rho^*}}\frac{\partial\tilde{c}_{\rho^*}}{\partial\rho^*}\frac{\partial\rho^*}{\partial\lambda_2}\right] &= 0\end{aligned} \tag{19}$$

Now recall from equation (17) that, since $h(\rho^*) < 0$, it must hold that $E[w_1] - h(\rho) > 0$ Thus, the second term in equation (19) must be negative. Since the first derivative is negative, the second is positive and $\lambda_2 < 1$ it must hold that

$$\frac{\partial\rho^*}{\partial\lambda_2} > 0$$

The second part results from taking the derivative of the ex-ante probability of proposing 1 with respect to λ_2 which is:

$$(1-\lambda_1)\left[\left(1-\int_{\underline{c}}^{\overline{c}} F_1(\tilde{c}_2)\ \mathrm{d}F_2(c_2)\right)+\int_{\underline{c}}^{\overline{c}} f_1(\tilde{c}_{\rho^*})\left(\frac{\partial \tilde{c}_{\rho^*}}{\partial \rho^*}\frac{\partial \rho^*}{\partial \lambda_2}\right)\ \mathrm{d}F_2(c_2)\right] \geq 0. \tag{20}$$

Within the parenthesis the first term is (weakly) positive since the conditional probability cannot be greater than 1. The second part is positive in all derivatives which makes the whole equation positive. Thus, the probability of proposing project 1 increases in λ_2.

To see the effects of a change in λ_2 on the RCCM, recall first the last term of equation (5) and observe that we may write this part as

$$(1-\lambda_2)g(\rho^*) := (1-\delta)(1-\lambda_2)(1-\lambda_1)\int_{\underline{c}}^{\overline{c}} w_2(c_2)F_1(\tilde{c}_{\rho^*}(c_2))\mathrm{d}F_2(c_2),$$

where $g(\rho^*)$ only depends indirectly (via ρ^*) on λ_2.
using $\phi(\rho^*)$ as defined above this enables us to state the following

$$\phi(\rho^*)E[w|0] = (1-\lambda_2)g(\rho^*)$$

dividing by $1-\lambda_2$ this straightforwardly yields

$$g(\rho^*) = \phi(\rho^*)\frac{E[w|0]}{(1-\lambda_2)}$$

Observe now that $\frac{E[w|0]}{(1-\lambda_2)}$ is constant over both λ_2 and ρ_2 and ϕ is thus proportional to g w.r.t. ρ^* (and λ_2).

Knowing that an increase λ_2 leads to an increase in ρ^* and thus an increase in $\tilde{c}_{\rho^*}(\cdot)$ also $g(\rho^*)$, and therefore $\phi(\rho^*)$ increases in ρ^*.

□

A.11 Proof of Lemma 8

Proof. First, recall from Lemma 6 that in the decision rule of the firm about which project to propose (in general) is the same in both the ρ^*-equilibrium and the waiting equilibrium at $\underline{\tau}$. Hence, the function $\tilde{c}_{\rho^*} = \hat{c}_{\underline{\tau}}$. Further, recall, that for any c_2 such that $\hat{c}_{\underline{\tau}}(c_2) \neq \{\overline{c}, \underline{c}\}$ the following equation holds

$$g\left(\delta, \underline{\tau}, \hat{c}_{\underline{\tau}}(c_2)\right) := \delta^{\underline{\tau}}\ \pi_1\left(\hat{c}_{\underline{\tau}}(c_2)\right) - \pi_2(c_2) = 0$$

Since expectations are positive, there is at least one state c_2 for which $g(\delta, \underline{\tau}, \hat{c}_{\underline{\tau}}(c_2)) = 0$. Next, totally differentiate $g(\cdot)$ to get

$$
\begin{aligned}
& \mathrm{d}g(\cdot) = 0 \\
\Leftrightarrow \quad & 0 = \left(\underline{\tau}\ \delta^{\underline{\tau}-1}\ \pi_1\left(\hat{c}_{\underline{\tau}}(c_2)\right)\right. && \mathrm{d}\delta \\
& \quad + \delta^{\underline{\tau}}\ \ln(\delta)\ \pi_1\left(\hat{c}_{\underline{\tau}}(c_2)\right) && \mathrm{d}\underline{\tau} \\
& \quad + \delta^{\underline{\tau}}\ \pi_1'\left(\hat{c}_{\underline{\tau}}(c_2)\right) && \mathrm{d}\hat{c}_{\underline{\tau}}(c_2) \\
\Leftrightarrow \quad & \frac{\mathrm{d}\underline{\tau}}{\mathrm{d}\delta} = \frac{-1}{\ln(d)}\left[\frac{\underline{\tau}}{\delta} + \frac{\pi_1'\left(\hat{c}_{\underline{\tau}}\right)}{\pi_1\left(\hat{c}_{\underline{\tau}}\right)}\frac{\mathrm{d}\hat{c}_{\underline{\tau}}}{\mathrm{d}\delta}\right]
\end{aligned}
$$

Since $\hat{c}_{\underline{\tau}} = \tilde{c}_{\rho^*}$ for all δ the following also holds

$$\frac{\mathrm{d}\hat{c}_{\underline{\tau}}}{\mathrm{d}\delta} = \frac{\mathrm{d}\tilde{c}_{\rho^*}}{\mathrm{d}\delta} \leq 0$$

as discussed in the proof of Proposition 2. Since $\pi_1' < 0$ by definition the term in square brackets is positive, and so is the one outside the brackets as $\delta < 1$. Thus,

$$\frac{\mathrm{d}\underline{\tau}}{\mathrm{d}\delta} > 0.$$

□

A.12 Proof of Proposition 3

Proof. The proof proceeds in three steps. First I show that the AA has a lower payoff in the waiting equilibrium at $\underline{\tau}$ than in the ρ^*-equilibrium. Second I show that if $T-1 \geq \overline{\tau}$, the waiting equilibrium at $T-1$ yields a higher payoff for the AA than the ρ^*-equilibrium and that the AA's waiting equilibrium payoffs are continuous in τ. Finally, I show that the derivative of the AA's payoff at the waiting equilibrium at $\overline{\tau}$ is in fact negative (and remains negative for all $T-1 \geq \overline{\tau}$) and thus, the "AA-most-preferred" equilibrium lies in the interior if T> $\overline{\tau}$.

First Step The claim in this part is, that $E_a[w|\rho^*] > E_a[w|\tau = \underline{\tau}]$, where $E[w|\tau]$ is the ex-ante payoff the AA expects in an waiting equilibrium at τ. The derivation of this claim is pretty straightforward. Recall that the decision rule in the two equilibria is nearly the same, only that in all cases in which the firm does not propose project 2 in both equilibria the firm chooses to wait in the waiting equilibrium while she directly proposes project 1 in the ρ^* equilibrium. In those cases the decision by the authority is again the same, once project 1 has been proposed. She is (interim) indifferent. Thus whenever the the AA sees a proposal of project 1 she knows at time t when message one is sent that

$$E_t[w_1|m_t = 1] = \delta E_t[w_2|m_t = 1].$$

However, in the waiting equilibrium we have $t = \underline{\tau}$ while in the ρ^*-equilibrium $t = 1$. Thus, ex-ante expected payoffs for the AA are

$$E_a[w|\rho^*] = E_a[w|0] - (1-\delta) \int_{\underline{c}}^{\bar{c}} \tilde{r}_{\rho^*}(c_2)\, \mathrm{d}F_2(c_2) \tag{21}$$

$$E_a[w|\tau = \underline{\tau}] = E_a[w|0] - (1-\delta^{\underline{\tau}+1}) \int_{\underline{c}}^{\bar{c}} \hat{r}_{\underline{\tau}}(c_2)\mathrm{d}F_2(c_2) \tag{22}$$

where

$$\tilde{r}_{\rho^*}(c_2) = (1-\lambda_2)(1-\lambda_1) \int_{\underline{c}}^{\bar{c}} w_2(c_2)F_1(\tilde{c}_{\rho^*}(c_2))\mathrm{d}F_2(c_2)$$

and $\hat{r}_{\underline{\tau}}(c_2)$ is defined respectively. Since $\tilde{c}_{\rho^*} = \hat{c}_{\underline{\tau}}$, the two expressions are identical except for the factor in front of the last term. Since $\underline{\tau} > 0$ by construction, in particular by assumption 4, (21)> (22) and the waiting equilibrium at $\underline{\tau}$ can never be optimal.

Second Step To see the second part, observe that when $T - 1 \geq \overline{\tau}$, it must be the case that under a waiting equilibrium at $\overline{\tau}$, the decision rule $\hat{c}_{\overline{\tau}}(c_2) = \underline{c}$ for all $c_2 \neq \bar{\bar{c}}$. In other words, whenever project 2 is available, the firm proposes it already in the first period. Thus, the AA gains profits of at least $E[w_2]$. If project 2 is not available, the firm still can make some profit by proposing project 1 after a waiting time of $T - 1$ periods. Thus, the AA's ex-ante payoff is

$$E[w|\tau = \overline{\tau}] = (1-\lambda_2)E[w_2] + \delta^{\overline{\tau}}\lambda_2(1-\lambda_1)E[w_1]. \tag{23}$$

This way, it is clear, that $E[w|\rho^*] < (1-\lambda_2)E[w_2]$.

Continuity can be shown, by observing that $\hat{c}_\tau(c_2)$ is continuous in τ for all c_2 since δ^τ is. Then, each part in the additive form of the ex-ante expectations of the AAs welfare is a continuous in τ and by that the expectations itself are as well.

Third Step Finally, I show that the derivative of the expected welfare for the AA at $\overline{\tau}$ is negative and remains negative as τ increases.

The latter is easy to see by inspecting equation (23). The first term is independent of τ while the second decreases in τ. As a consequence, each waiting equilibrium at $t \geq \overline{\tau}$ yields lower welfare than the waiting equilibrium at $\overline{\tau}$. If projects are such that $\pi_1(\underline{c}) \geq \pi_2(\bar{c}$, then it is even possible to find a neighbourhood of size ϵ around τ such that the "AA-most-preferred" waiting equilibrium is in fact shorter than $\overline{\tau}$.

□

A.13 Proof of Proposition 4

Proof. To begin with, observe that, if the AA would only play pure strategies, delaying a decision is not optimal. I first prove that a rule at which only project two is accepted cannot be optimal. The proof can w.l.o.g. be applied to a situation where the candidate is the other way around, that is 1 gets always accepted and 2 never. In the second part I show that an acceptance vector of (1,1) cannot be optimal either.

First Part In this part, I show that it is never optimal for the AA to commit to a strategy in which project 2 is always accepted and project 1 is always rejected.

First, recall that in such an equilibrium the ex-ante expectation of the AA were $(1-\lambda_2)E[w_2]$ since the firm always proposes 2 if it is available and something else in all other cases, but only two gets accepted.

Second, suppose now that the AA accepts project 1 with a probability $\rho_1^1 = \epsilon$, where ϵ is chosen such that

$$\epsilon\pi_1(\underline{c}) + (1-\epsilon)\delta\pi_2(\overline{c}) < \pi_2(\overline{c}). \tag{24}$$

From the discussion in section 4 we know that such an ϵ exists (since $\pi_i(\overline{c}) > 0$) and the firm would under this probability rule still always propose 2 whenever it is available. However, if only 1 was available the firm would actually strictly prefer to propose 1.

The ex-ante expected value of such a probability rule for the AA would be

$$(1-\lambda_2)E[w_2] + \epsilon\lambda_2(1-\lambda_1)E[w_1] > (1-\lambda_2)E[w_2]. \tag{25}$$

Thus, it cannot be optimal to accept 2 always and never 1.

Second Part In this part I, show that a "accept all" policy is even worse than what we derived in the first part. This follows, essentially by definition and is repeated here only for the sake of completeness. Suppose the firm accepts all proposals in the first round. If the AA were to choose an acceptance probability of 1 for both projects, the firm would propose naively. Thus the ex-ante expected payoff for the AA was

$$E[w]_{(1,1)} := P_{\pi_1>\pi_2}E[w_1|\pi_1 > \pi_2] + P_{\pi_2>\pi_2}E[w_2|\pi_2 > \pi_1], \tag{26}$$

where $P_{\pi_i>\pi_j}$ denotes the probability that $\pi_i > \pi_j$.

Now recall the payoff the AA earns when only commiting to project two, that is

$$(1-\lambda_2)E[w_2], \tag{27}$$

By assumption 4 it holds that

$$E[w]_{(1,1)} < P_{\pi_1>\pi_2}\delta E[w_2|\pi_1 > \pi_2] + P_{\pi_2>\pi_2}E[w_2|\pi_2 > \pi_1].$$

But then (26)<(27) or in other words, “accept all” is ex-ante less profitable for the AA than “accept only 2”.

Finally, accepting no project at all is not optimal since it has no influence on the behaviour of the firm, but leads only to waiting costs for both parties.

□